1. Bond Yields. A 30 -year Treasury bond is issued with face value of $\$ 1,000$, paying interest of $\$ 60$ per year. If market yields increase shortly after the T-bond is issued, what happens to the bond's
a. coupon rate? (LOI)
b. price? (LOI)
c. yield to maturity? (LOI)
d. current yield? (LOI)
2. Bond Yields. If a bond with face value of $\$ 1,000$ and a coupon rate of $8 \%$ is selling at a price of $\$ 970$, is the bond's yield to maturity more or less than $8 \%$ ? What about the current yield? (LO1)
3. Bond Yields. A bond with face value $\$ 1,000$ has a current yield of $7 \%$ and a coupon rate of $8 \%$. What is the bond's price? (LOI)
4. Bond Pricing. A 6-year Circular File bond pays interest of $\$ 80$ annually and sells for $\$ 950$. What are its coupon rate, current yield, and yield to maturity? (LO2)
5. Bond Pricing. If Circular File (see Quiz Question 4) wants to issue a new 6-year bond at face value, what coupon rate must the bond offer? (LO2)
6. Bond Yields. A bond has 10 years until maturity, a coupon rate of $8 \%$, and sells for $\$ 1,100$.
a. What is the current yield on the bond? (LOI)
b. What is the yield to maturity? (LO2)
7. Coupon Rate. General Matter's outstanding bond issue has a coupon rate of $10 \%$ and a current yield of $9.6 \%$, and it sells at a yield to maturity of $9.25 \%$. The firm wishes to issue additional bonds to the public at face value. What coupon rate must the new bonds offer in order to sell at face value? ( LO 2 )
8. Financial Pages. Turn back to Table 6-1. What is the current yield of the $7.125 \% 2023$ maturity bond? Why is this more than its yield to maturity? (LOI)
9. Bond Prices and Returns. One bond has a coupon rate of $8 \%$, another a coupon rate of $12 \%$. Both bonds have 10 -year maturities and sell at a yield to maturity of $10 \%$. If their yields to maturity next year are still $10 \%$, what is the rate of return on each bond? Does the higher coupon bond give a higher rate of return? (LO2)
10. Bond Returns. (LO2)
a. If the bond in Quiz Question 6 has a yield to maturity of $8 \% 1$ year from now, what will its price be?
b. What will be the rate of return on the bond?
c. If the inflation rate during the year is $3 \%$, what is the real rate of return on the bond?
11. Bond Pricing. A General Motors bond carries a coupon rate of $8 \%$, has 9 years until maturity, and sells at a yield to maturity of $7 \%$.
a. What interest payments do bondholders receive each year? (LO1)
b. At what price does the bond sell? (Assume annual interest payments.) (LO2)
c. What will happen to the bond price if the yield to maturity falls to $6 \%$ ? (LO2)
12. Bond Pricing. A 30 -year maturity bond with face value of $\$ 1,000$ makes annual coupon payments and has a coupon rate of $8 \%$. What is the bond's yield to maturity if the bond is selling for
a. $\$ 900$ ( LO 2 )
b. $\$ 1,000$ ? (LO2)
c. $\$ 1,100$ ? ( LO 2 )
13. Bond Pricing. Repeat the previous problem assuming semiannual coupon payments. (LO2)
14. Bond Pricing. Fill in the table below for the following zero-coupon bonds. The face value of each bond is $\$ 1,000$. (LO2)

| Price | Maturity (years) | Yield to Maturity |
| :---: | :---: | :---: |
| $\$ 300$ | 30 | - |
| 300 | - | $8 \%$ |
| - | 10 | 10 |

15. Consol Bonds. Perpetual Life Corp. has issued consol bonds with coupon payments of $\$ 60$. (Consols pay interest forever and never mature. They are perpetuities.) If the required rate of return on these bonds at the time they were issued was $6 \%$, at what price were they sold to the public? If the required return today is $10 \%$, at what price do the consols sell? (LO2)
16. Bond Pricing. Sure Tea Co. has issued $9 \%$ annual coupon bonds that are now selling at a yield to maturity of $10 \%$ and current yield of $9.8375 \%$. What is the remaining maturity of these bonds? (LO2)
17. Bond Pricing. Large Industries bonds sell for $\$ 1,065.15$. The bond life is 9 years, and the yield to maturity is $7 \%$. What must be the coupon rate on the bonds? (LO2)
18. Bond Prices and Yields.
a. Several years ago, Castles in the Sand, Inc., issued bonds at face value at a yield to maturity of $7 \%$. Now, with 8 years left until the maturity of the bonds, the company has run into hard
times and the yield to maturity on the bonds has increased to $15 \%$. What has happened to the price of the bond? (LO2)
b. Suppose that investors believe that Castles can make good on the promised coupon payments, but that the company will go bankrupt when the bond matures and the principal comes due. The expectation is that investors will receive only $80 \%$ of face value at maturity. If they buy the bond today, what yield to maturity do they expect to receive? (LO4)
19. Bond Returns. You buy an $8 \%$ coupon, 10-year maturity bond for $\$ 980$. A year later, the bond price is $\$ 1,100$. (LO2)
a. What is the new yield to maturity on the bond?
b. What is your rate of return over the year?
20. Bond Returns. You buy an $8 \%$ coupon, 20-year maturity bond when its yield to maturity is $9 \%$. A year later, the yield to maturity is $10 \%$. What is your rate of return over the year? (LO3)
21. Interest Rate Risk. Consider three bonds with $8 \%$ coupon rates, all selling at face value. The short-term bond has a maturity of 4 years, the intermediate-term bond has maturity 8 years, and the long-term bond has maturity 30 years. (LO3)
a. What will happen to the price of each bond if their yields increase to $9 \%$ ?
b. What will happen to the price of each bond if their yields decrease to $7 \%$ ?
c. What do you conclude about the relationship between time to maturity and the sensitivity of bond prices to interest rates?
22. Rate of Return. A 2-year maturity bond with face value of $\$ 1,000$ makes annual coupon payments of $\$ 80$ and is selling at face value. What will be the rate of return on the bond if its yield to maturity at the end of the year is
a. $6 \%$ ? (LO3)
b. $8 \%$ ? (LO3)
c. $10 \%$ ? (LO3)
23. Rate of Return. A bond that pays coupons annually is issued with a coupon rate of $4 \%$, maturity of 30 years, and a yield to maturity of $7 \%$. What rate of return will be earned by an investor who purchases the bond and holds it for 1 year if the bond's yield to maturity at the end of the year is $8 \%$ ? (LO3)
24. Credit Risk. A bond's credit rating provides a guide to its risk. Long-term bonds rated Aa currently offer yields to maturity of $7.5 \%$. A-rated bonds sell at yields of $7.8 \%$. If a 10 -year bond with a coupon rate of $7 \%$ is downgraded by Moody's from Aa to A rating, what is the likely effect on the bond price? (LO4)
25. Real Returns. Suppose that you buy a 1-year maturity bond for $\$ 1,000$ that will pay you back $\$ 1,000$ plus a coupon payment of $\$ 60$ at the end of the year. What real rate of return will you earn if the inflation rate is
a. $2 \%$ ? (LO3)
b. $4 \%$ ? (LO3)
c. $6 \%$ ? (LO3)
d. $8 \%$ ? (LO3)
26. Real Returns. Now suppose that the bond in the previous problem is a TIPS (inflation-indexed) bond with a coupon rate of $4 \%$. What will the cash flow provided by the bond be for each of the four inflation rates? What will be the real and nominal rates of return on the bond in each scenario? (LOI)
27. Real Returns. Now suppose the TIPS bond in the previous problem is a 2-year maturity bond. What will be the bondholder's cash flows in each year in each of the inflation scenarios? (LOI)
28. Interest Rate Risk. Suppose interest rates increase from $8 \%$ to $9 \%$. Which bond will suffer the greater percentage decline in price: a 30 -year bond paying annual coupons of $8 \%$ or a 30 -year
zero-coupon bond? Can you explain intuitively why the zero exhibits greater interest rate risk even though it has the same maturity as the coupon bond? (LO3)
29. Interest Rate Risk. Consider two 30 -year maturity bonds. Bond A has a coupon rate of $4 \%$, while bond B has a coupon rate of $12 \%$. Both bonds pay their coupons semiannually. (LO3)
a. Construct an Excel spreadsheet showing the prices of each of these bonds for yields to maturity ranging from $2 \%$ to $15 \%$ at intervals of $1 \%$. Column A should show the yield to maturity (ranging from $2 \%$ to $15 \%$ ), and columns B and C should compute the prices of the two bonds (using Excel's bond price function) at each interest rate.
b. In columns D and E, compute the percentage difference between the bond price and its value when yield to maturity is $8 \%$.
c. Plot the values in columns D and E as a function of the interest rate. Which bond's price is proportionally more sensitive to interest rate changes?
d. Can you explain the result you found in part (c)? Hint: Is there any sense in which a bond that pays a high coupon rate has lower "average" or "effective" maturity than a bond that pays a low coupon rate?
30. Yield Curve. In Figure 6-7, we saw a plot of the yield curve on stripped Treasury bonds and pointed out that bonds of different maturities may sell at different yields to maturity. In principle, when we are valuing a stream of cash flows, each cash flow should be discounted by the yield appropriate to its particular maturity. Suppose the yield curve on (zero-coupon) Treasury strips is as follows:

| Time to Maturity | YTM |
| :--- | :--- |
| 1 year | $4.0 \%$ |
| 2 | 5.0 |
| $3-5$ | 5.5 |
| $6-10$ | 6.0 |

You wish to value a 10 -year bond with a coupon rate of $10 \%$, paid annually. (LO2)
a. Set up an Excel spreadsheet to value each of the bond's annual cash flows using this table of yields. Add up the present values of the bond's 10 cash flows to obtain the bond price.
b. What is the bond's yield to maturity?
c. Compare the yield to maturity of the 10 -year, $10 \%$ coupon bond to that of a 10 -year zerocoupon bond or Treasury strip. Which is higher? Why does this result make sense given this yield curve?

## Solutions to Chapter 6 <br> Valuing Bonds

1. a. Coupon rate $=6 \%$, which remains unchanged. The coupon payments are fixed at $\$ 60$ per year.
b. When the market yield increases, the bond price will fall. The cash flows are discounted at a higher rate.
c. At a lower price, the bond's yield to maturity will be higher. The higher yield to maturity for the bond is commensurate with the higher yields available in the rest of the bond market.
d. Current yield = coupon rate/bond price

As coupon rate remains the same and the bond price decreases, the current yield increases.
2. When the bond is selling at a discount, $\$ 970$ in this case, the yield to maturity is greater than $8 \%$. We know that if the yield to maturity were $8 \%$, the bond would sell at par. At a price below par, the yield to maturity exceeds the coupon rate.

Current yield $=$ coupon payment/bond price $=\$ 80 / \$ 970$
Therefore, current yield is also greater than $8 \%$.
3. Coupon payment $=0.08 \times \$ 1,000=\$ 80$

Current yield $=\$ 80 /$ bond price $=0.07$
Therefore: bond price $=\$ 80 / 0.07=\$ 1,142.86$
4. Coupon rate $=\$ 80 / \$ 1,000=0.080=8.0 \%$

Current yield $=\$ 80 / \$ 950=0.0842=8.42 \%$
To compute the yield to maturity, use trial and error to solve for $r$ in the following equation:

$$
\$ 950=\$ 80 \times\left[\frac{1}{r}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{6}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{6}} \Rightarrow \mathrm{r}=9.119 \%
$$

Using a financial calculator, compute the yield to maturity by entering:
$\mathrm{n}=6 ; \mathrm{PV}=(-) 950 ; \mathrm{FV}=1000 ; \mathrm{PMT}=80$, compute $\mathrm{i}=9.119 \%$

Verify the solution as follows:

$$
\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.09119}-\frac{1}{0.09119(1.09119)^{6}}\right]+\frac{\$ 1,000}{1.09119^{6}}=\$ 949.98
$$

(difference due to rounding)
5. In order for the bond to sell at par, the coupon rate must equal the yield to maturity. Since Circular bonds yield $9.119 \%$, this must be the coupon rate.
6. a. Current yield $=$ coupon $/$ price $=\$ 80 / \$ 1,100=0.0727=7.27 \%$
b. To compute the yield to maturity, use trial and error to solve for $r$ in the following equation:

$$
\$ 1,100=\$ 80 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{10}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{10}} \Rightarrow \mathrm{r}=6.602 \%
$$

Using a financial calculator, compute the yield to maturity by entering: $\mathrm{n}=10 ; \mathrm{PV}=(-) 1100 ; \mathrm{FV}=1000 ; \mathrm{PMT}=80$, compute $\mathrm{i}=6.602 \%$

Verify the solution as follows:

$$
\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.06602}-\frac{1}{0.06602(1.06602)^{10}}\right]+\frac{\$ 1,000}{1.06602^{10}}=\$ 1,100.02
$$

(difference due to rounding)
7. When the bond is selling at face value, its yield to maturity equals its coupon rate. This firm's bonds are selling at a yield to maturity of $9.25 \%$. So the coupon rate on the new bonds must be $9.25 \%$ if they are to sell at face value.
8. The bond pays a coupon of $7.125 \%$ which means annual interest is $\$ 71.25$. The bond is selling for: $1305 / 32=\$ 1,301.5625$. Therefore, the current yield is: $\$ 71.25 / \$ 1301.5625=5.47 \%$
The current yield exceeds the yield-to-maturity on the bond because the bond is selling at a premium. At maturity the holder of the bond will receive only the $\$ 1,000$ face value, reducing the total return on investment.
9. Bond 1

Year 1: $\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.10}-\frac{1}{0.10(1.10)^{10}}\right]+\frac{\$ 1,000}{1.10^{10}}=\$ 877.11$
Year 2: $\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.10}-\frac{1}{0.10(1.10)^{9}}\right]+\frac{\$ 1,000}{1.10^{9}}=\$ 884.82$
Using a financial calculator:
Year 1: $\quad \mathrm{PMT}=80, \mathrm{FV}=1000, \mathrm{i}=10 \%, \mathrm{n}=10$; compute $\mathrm{PV}_{0}=\$ 877.11$
Year 2: $\quad \mathrm{PMT}=80, \mathrm{FV}=1000, \mathrm{i}=10 \%, \mathrm{n}=9$; compute $\mathrm{PV}_{1}=\$ 884.82$
Rate of return $=\frac{\$ 80+(\$ 884.82-\$ 877.11)}{\$ 877.11}=0.100=10.0 \%$
Bond 2
Year 1: $\mathrm{PV}=\$ 120 \times\left[\frac{1}{0.10}-\frac{1}{0.10(1.10)^{10}}\right]+\frac{\$ 1,000}{1.10^{10}}=\$ 1,122.89$
Year 2: PV $=\$ 120 \times\left[\frac{1}{0.10}-\frac{1}{0.10(1.10)^{9}}\right]+\frac{\$ 1,000}{1.10^{9}}=\$ 1,115.18$
Using a financial calculator:
Year 1: $\quad \mathrm{PMT}=120, \mathrm{FV}=1000, \mathrm{i}=10 \%, \mathrm{n}=10 ;$ compute $\mathrm{PV}_{0}=\$ 1,122.89$
Year 2: $\quad \mathrm{PMT}=120, \mathrm{FV}=1000, \mathrm{i}=10 \%, \mathrm{n}=9$; compute $\mathrm{PV} 1=\$ 1,115.18$
Rate of Return $=\frac{\$ 120+(\$ 1,115.18-\$ 1,122.89)}{\$ 1,122.89}=0.100=10.0 \%$
Both bonds provide the same rate of return.
10. a. If yield to maturity $=8 \%$, price will be $\$ 1,000$.
b. Rate of return $=$

$$
\frac{\text { coupon income }+ \text { price change }}{\text { investment }}=\frac{\$ 80+(\$ 1,000-\$ 1,100)}{\$ 1,100}=-0.0182=-1.82 \%
$$

c. Real return $=$ Error $!-1=\frac{0.9818}{1.03}-1=-0.0468=-4.68 \%$
11. a. With a par value of $\$ 1,000$ and a coupon rate of $8 \%$, the bondholder receives $\$ 80$ per year.
b. $\quad \mathrm{PV}=\$ 80 \times\left[\frac{1}{0.07}-\frac{1}{0.07 \times(1.07)^{9}}\right]+\frac{\$ 1,000}{(1.07)^{9}}=\$ 1,065.15$
c. If the yield to maturity is $6 \%$, the bond will sell for:

$$
\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.06}-\frac{1}{0.06 \times(1.06)^{9}}\right]+\frac{\$ 1,000}{(1.06)^{9}}=\$ 1,136.03
$$

12. a. To compute the yield to maturity, use trial and error to solve for $r$ in the following equation:

$$
\$ 900=\$ 80 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{30}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{30}} \Rightarrow \mathrm{r}=8.971 \%
$$

Using a financial calculator, compute the yield to maturity by entering: $\mathrm{n}=30 ; \mathrm{PV}=(-) 900 ; \mathrm{FV}=1000 ; \mathrm{PMT}=80$, compute $\mathrm{i}=8.971 \%$

Verify the solution as follows:

$$
\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.08971}-\frac{1}{0.08971(1.08971)^{30}}\right]+\frac{\$ 1,000}{1.08971^{30}}=\$ 899.99
$$

(difference due to rounding)
b. Since the bond is selling for face value, the yield to maturity $=8.000 \%$
c. To compute the yield to maturity, use trial and error to solve for $r$ in the following equation:

$$
\$ 1,100=\$ 80 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{30}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{30}} \Rightarrow \mathrm{r}=7.180 \%
$$

Using a financial calculator, compute the yield to maturity by entering: $\mathrm{n}=30 ; \mathrm{PV}=(-) 1100 ; \mathrm{FV}=1000 ; \mathrm{PMT}=80$, compute $\mathrm{i}=7.180 \%$
Verify the solution as follows:

$$
\mathrm{PV}=\$ 80 \times\left[\frac{1}{0.07180}-\frac{1}{0.07180(1.07180)^{30}}\right]+\frac{\$ 1,000}{1.07180^{30}}=\$ 1,099.94
$$

(difference due to rounding)
13. a. To compute the yield to maturity, use trial and error to solve for $r$ in the following equation:

$$
\$ 900=\$ 40 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{60}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{60}} \Rightarrow \mathrm{r}=4.483 \%
$$

Using a financial calculator, compute the yield to maturity by entering:
$\mathrm{n}=60 ; \mathrm{PV}=(-) 900 ; \mathrm{FV}=1000 ; \mathrm{PMT}=40$, compute $\mathrm{i}=4.483 \%$
Verify the solution as follows:

$$
\mathrm{PV}=\$ 40 \times\left[\frac{1}{0.04483}-\frac{1}{0.04483(1.04483)^{60}}\right]+\frac{\$ 1,000}{1.04483^{60}}=\$ 900.02
$$

(difference due to rounding)
Therefore, the annualized bond equivalent yield to maturity is:

$$
4.483 \% \times 2=8.966 \%
$$

b. Since the bond is selling for face value, the semi-annual yield $=4 \%$

Therefore, the annualized bond equivalent yield to maturity is: $4 \% \times 2=8 \%$
c. To compute the yield to maturity, use trial and error to solve for $r$ in the following equation:

$$
\$ 1,100=\$ 40 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{60}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{60}} \Rightarrow \mathrm{r}=3.592 \%
$$

Using a financial calculator, compute the yield to maturity by entering: $\mathrm{n}=60 ; \mathrm{PV}=(-) 1100 ; \mathrm{FV}=1000 ; \mathrm{PMT}=40$, compute $\mathrm{i}=3.592 \%$
Verify the solution as follows:

$$
\mathrm{PV}=\$ 40 \times\left[\frac{1}{0.03592}-\frac{1}{0.03592(1.03592)^{60}}\right]+\frac{\$ 1,000}{1.03592^{60}}=\$ 1,099.92
$$

(difference due to rounding)
Therefore, the annualized bond equivalent yield to maturity is:

$$
3.592 \% \times 2=7.184 \%
$$

14. In each case, we solve the following equation for the missing variable:

$$
\text { Price }=\$ 1,000 /(1+y)^{\text {maturity }}
$$

| Price | Maturity (Years) | Yield to Maturity |
| :---: | :---: | :---: |
| $\$ 300.00$ | 30.00 | $4.095 \%$ |
| $\$ 300.00$ | 15.64 | $8.000 \%$ |
| $\$ 385.54$ | 10.00 | $10.000 \%$ |

15. PV of perpetuity $=$ coupon payment/rate of return.
$\mathrm{PV}=\mathrm{C} / \mathrm{r}=\$ 60 / 0.06=\$ 1,000.00$
If the required rate of return is $10 \%$, the bond sells for:
$\mathrm{PV}=\mathrm{C} / \mathrm{r}=\$ 60 / 0.10=\$ 600.00$
16. Current yield $=0.098375$ so bond price can be solved from the following:

$$
\$ 90 / \text { Price }=0.098375 \Rightarrow \text { Price }=\$ 914.87
$$

To compute the remaining maturity, solve for t in the following equation:

$$
\$ 914.87=\$ 90 \times\left[\frac{1}{0.10}-\frac{1}{0.10 \times(1.10)^{\mathrm{t}}}\right]+\frac{\$ 1,000}{(1.10)^{t}} \Rightarrow \mathrm{t}=20.0
$$

Using a financial calculator, compute the remaining maturity by entering: $\mathrm{PV}=(-) 914.87 ; \mathrm{FV}=1000 ; \mathrm{PMT}=90, \mathrm{i}=10$ and compute $\mathrm{n}=20.0$ years.
17. Solve the following equation for PMT:

$$
\$ 1,065.15=\mathrm{PMT} \times\left[\frac{1}{0.07}-\frac{1}{0.07 \times(1.07)^{9}}\right]+\frac{\$ 1,000}{(1.07)^{9}} \Rightarrow \mathrm{PMT}=\$ 80.00
$$

Using a financial calculator, compute the annual payment by entering:
$\mathrm{n}=9 ; \mathrm{PV}=(-) 1065.15 ; \mathrm{FV}=1000 ; \mathrm{i}=7$, compute $\mathrm{PMT}=\$ 80.00$
Since the annual payment is $\$ 80$, the coupon rate is $8 \%$.
18. a. The coupon rate must be $7 \%$ because the bonds were issued at face value with a yield to maturity of $7 \%$. Now, the price is:

$$
\mathrm{PV}=\$ 70 \times\left[\frac{1}{0.15}-\frac{1}{0.15(1.15)^{8}}\right]+\frac{\$ 1,000}{1.15^{8}}=\$ 641.01
$$

b. The investors pay $\$ 641.01$ for the bond. They expect to receive the promised coupons plus $\$ 800$ at maturity. We calculate the yield to maturity based on these expectations by solving the following equation for r :

$$
\$ 641.01=\$ 70 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{8}}\right]+\frac{\$ 800}{(1+\mathrm{r})^{8}} \Rightarrow \mathrm{r}=12.87 \%
$$

Using a financial calculator, enter: $\mathrm{n}=8 ; \mathrm{PV}=(-) 641.01 ; \mathrm{FV}=800 ; \mathrm{PMT}=70$, and then compute $\mathrm{i}=12.87 \%$
19. a. At a price of $\$ 1,100$ and remaining maturity of 9 years, find the bond's yield to maturity by solving for $r$ in the following equation:

$$
\$ 1,100=\$ 80 \times\left[\frac{1}{\mathrm{r}}-\frac{1}{\mathrm{r} \times(1+\mathrm{r})^{9}}\right]+\frac{\$ 1,000}{(1+\mathrm{r})^{9}} \Rightarrow \mathrm{r}=6.50 \%
$$

Using a financial calculator, enter: $\mathrm{n}=9 ; \mathrm{PV}=(-) 1100 ; \mathrm{FV}=1000 ; \mathrm{PMT}=80$, and then compute $\mathrm{i}=6.50 \%$
b. $\quad$ Rate of return $=\frac{\$ 80+(\$ 1,100-\$ 980)}{\$ 980}=20.41 \%$
20. $\mathrm{PV}_{0}=\$ 80 \times\left[\frac{1}{0.09}-\frac{1}{0.09(1.09)^{20}}\right]+\frac{\$ 1,000}{1.09^{20}}=\$ 908.71$
$\mathrm{PV}_{1}=\$ 80 \times\left[\frac{1}{0.10}-\frac{1}{0.10(1.10)^{19}}\right]+\frac{\$ 1,000}{1.10^{19}}=\$ 832.70$
Rate of return $=\frac{\$ 80+(\$ 832.70-\$ 908.71)}{\$ 908.71}=0.0044=0.44 \%$
21. $\mathrm{a}, \mathrm{b}$.

Price of each bond at different yields to maturity

|  | Maturity of bond |  |  |
| :---: | :---: | :---: | :---: |
|  | 4 years | 8 years | 30 years |
| Yield |  |  |  |
| 7\% | \$1,033.87 | \$1,059.71 | \$1,124.09 |
| 8\% | \$1,000.00 | \$1,000.00 | \$1,000.00 |
| 9\% | \$967.60 | \$944.65 | \$897.26 |

c. The table shows that prices of longer-term bonds are more sensitive to changes in interest rates.
22. The price of the bond at the end of the year depends on the interest rate at that time. With one year until maturity, the bond price will be: $\$ 1,080 /(1+r)$
a. $\quad$ Price $=\$ 1,080 / 1.06=\$ 1,018.87$

Rate of Return $=[\$ 80+(\$ 1,018.87-\$ 1,000)] / \$ 1,000=0.0989=9.89 \%$
b. $\quad$ Price $=\$ 1,080 / 1.08=\$ 1,000.00$

Rate of Return $=[\$ 80+(\$ 1,000-\$ 1,000)] / \$ 1,000=0.0800=8.00 \%$
c. $\quad$ Price $=\$ 1,080 / 1.10=\$ 981.82$

Rate of Return $=[\$ 80+(\$ 981.82-\$ 1,000)] / \$ 1000=0.0618=6.18 \%$
23. The original price of the bond is computed as follows:

$$
\mathrm{PV}=\$ 40 \times\left[\frac{1}{0.07}-\frac{1}{0.07(1.07)^{30}}\right]+\frac{\$ 1,000}{1.07^{30}}=\$ 627.73
$$

After one year, the maturity of the bond will be 29 years and its price will be:

$$
\mathrm{PV}=\$ 40 \times\left[\frac{1}{0.08}-\frac{1}{0.08(1.08)^{29}}\right]+\frac{\$ 1,000}{1.08^{29}}=\$ 553.66
$$

The capital loss on the bond is $\$ 74.07$. The rate of return is therefore:

$$
(\$ 40-\$ 74.07) / \$ 627.73=-0.0543=-5.43 \%
$$

24. The bond's yield to maturity will increase from $7.5 \%$ to $7.8 \%$ when the perceived default risk increases. The bond price will fall:

$$
\begin{aligned}
& \text { Initial Price }=P V=\$ 70 \times\left[\frac{1}{0.075}-\frac{1}{0.075(1.075)^{10}}\right]+\frac{\$ 1,000}{1.075^{10}}=\$ 965.68 \\
& \text { New Price }=P V=\$ 70 \times\left[\frac{1}{0.078}-\frac{1}{0.078(1.078)^{10}}\right]+\frac{\$ 1,000}{1.078^{10}}=\$ 945.83
\end{aligned}
$$

25. The nominal rate of return is $6 \%$.

The real rate of return is: $[1.06 /(1+$ inflation $)]-1$
a. $1.06 / 1.02-1=0.0392=3.92 \%$
b. $1.06 / 1.04-1=0.0192=1.92 \%$
c. $1.06 / 1.06-1=0.00=0 \%$
d. $1.06 / 1.08-1=-0.0185=-1.85 \%$
26. The principal value of the bond will increase by the inflation rate, and since the coupon is $4 \%$ of the principal, the coupon will also increase along with the general level of prices. The total cash flow provided by the bond will be:

$$
1000 \times(1+\text { inflation rate })+\text { coupon rate } \times 1000 \times(1+\text { inflation rate }) .
$$

Since the bond is purchased for face value, or $\$ 1,000$, total dollar nominal return is therefore the increase in the principal due to the inflation indexing, plus coupon income:

$$
\text { Income }=[\$ 1,000 \times \text { inflation rate }]+[\text { coupon rate } \times \$ 1,000 \times(1+\text { inflation rate })]
$$

Finally: nominal rate of return $=$ income $/ \$ 1,000$
a. $\quad$ Nominal rate of return $=\frac{\$ 20+(\$ 40 \times 1.02)}{\$ 1,000}=0.0608=6.08 \%$

$$
\text { Real rate of return }=\frac{1.0608}{1.02}-1=0.0400=4.00 \%
$$

b. Nominal rate of return $=\frac{\$ 40+(\$ 40 \times 1.04)}{\$ 1,000}=0.0816=8.16 \%$

$$
\text { Real rate of return }=\frac{1.0816}{1.04}-1=0.0400=4.00 \%
$$

c. Nominal rate of return $=\frac{\$ 60+(\$ 40 \times 1.06)}{\$ 1,000}=0.1024=10.24 \%$

$$
\text { Real rate of return }=\frac{1.1024}{1.06}-1=0.0400=4.00 \%
$$

d. Nominal rate of return $=\frac{\$ 80+(\$ 40 \times 1.08)}{\$ 1,000}=0.1232=12.32 \%$

$$
\text { Real rate of return }=\frac{1.1232}{1.08}-1=0.0400=4.00 \%
$$

27. 

|  | First year cash flow | Second year cash flow |
| :--- | :---: | :---: |
| a. | $\$ 40 \times 1.02=\$ 40.80$ | $\$ 1,040 \times 1.02^{2}=\$ 1,082.016$ |
| b. | $\$ 40 \times 1.04=\$ 41.60$ | $\$ 1,040 \times 1.04^{2}=\$ 1,124.864$ |
| c. | $\$ 40 \times 1.06=\$ 42.40$ | $\$ 1,040 \times 1.06^{2}=\$ 1,168.544$ |
| d. | $\$ 40 \times 1.08=\$ 43.20$ | $\$ 1,040 \times 1.08^{2}=\$ 1,213.056$ |

28. The coupon bond will fall from an initial price of $\$ 1,000$ (when yield to maturity $=8 \%$ ) to a new price of $\$ 897.26$ when yield to maturity immediately rises to $9 \%$. This is a $10.27 \%$ decline in the bond price.

The initial price of the zero-coupon bond is: $\frac{\$ 1,000}{1.08^{30}}=\$ 99.38$
The new price of the zero-coupon bond is: $\frac{\$ 1,000}{1.09^{30}}=\$ 75.37$
This is a price decline of $24.16 \%$, far greater than that of the coupon bond.
The price of the coupon bond is much less sensitive to the change in yield. It seems to act like a shorter maturity bond. This makes sense: there are many coupon payments for the $8 \%$ bond, most of which come years before the bond's maturity date. Each payment may be considered to have its own "maturity date" which suggests that the effective maturity of the bond should be measured as some sort of average of the maturities of all the cash flows paid out by the bond. The zero-coupon bond, by contrast, makes only one payment at the final maturity date.
29. a, b.

| Yield | Price A | Price B | \%diff(8\%) A | \%diff(8\%) B |
| :---: | ---: | ---: | ---: | ---: |
|  | 144.93 | 324.67 | $165 \%$ | $124 \%$ |
| $3 \%$ | 119.68 | 277.14 | $119 \%$ | $91 \%$ |
| $4 \%$ | 100.00 | 239.00 | $83 \%$ | $65 \%$ |
| $5 \%$ | 84.55 | 208.15 | $54 \%$ | $43 \%$ |
| $6 \%$ | 72.33 | 183.00 | $32 \%$ | $26 \%$ |
| $7 \%$ | 62.59 | 162.35 | $14 \%$ | $12 \%$ |
| $8 \%$ | 54.76 | 145.24 | $0 \%$ | $0 \%$ |
| $9 \%$ | 48.41 | 130.95 | $-12 \%$ | $-10 \%$ |
| $10 \%$ | 43.22 | 118.92 | $-21 \%$ | $-18 \%$ |
| $11 \%$ | 38.93 | 108.72 | $-29 \%$ | $-25 \%$ |
| $12 \%$ | 35.36 | 99.99 | $-35 \%$ | $-31 \%$ |
| $13 \%$ | 32.35 | 92.48 | $-41 \%$ | $-36 \%$ |
| $14 \%$ | 29.80 | 85.95 | $-46 \%$ | $-41 \%$ |
| $15 \%$ | 27.62 | 80.25 | $-50 \%$ | $-45 \%$ |
|  |  |  |  |  |
| c. |  |  |  |  |



The price of bond A is more sensitive to interest rate changes as reflected in the steeper curve.
d. Bond A has a higher effective maturity (higher duration). A bond that pays a high coupon rate has a lower effective maturity since a greater proportion of the total return to the investment is received before maturity. A bond that pays a lower coupon rate has a longer average time to each payment.
30. a, b.

| Year | YTM | Cash Flow from Bond | PV of Cash Flow |
| :---: | :---: | :---: | :---: |
| 1 | 4.0\% | 100 | 96.15384615 |
| 2 | 5.0\% | 100 | 90.70294785 |
| 3 | 5.5\% | 100 | 85.16136642 |
| 4 | 5.5\% | 100 | 80.72167433 |
| 5 | 5.5\% | 100 | 76.51343538 |
| 6 | 6.0\% | 100 | 70.49605404 |
| 7 | 6.0\% | 100 | 66.50571136 |
| 8 | 6.0\% | 100 | 62.74123713 |
| 9 | 6.0\% | 100 | 59.18984635 |
| 10 | 6.0\% | 1100 | 614.2342546 |
| $\begin{array}{lr} \text { Bond Price (PV) }= & 1302.420374 \\ \text { YTM (RATE) }= & 5.91 \% \end{array}$ |  |  |  |
|  |  |  |  |

c. The yield to maturity on the zero-coupon bond is higher. The zero-coupon has a higher effective maturity (higher duration) in that a greater proportion of the cash flow is received at the maturity. The zero-coupon bond is therefore more sensitive to changes in interest rates which are expected to rise based on this upward sloping yield curve.

