Problems 1–8 refer to two projects with the following cash flows:

Year	Project A	Project B
0	−$200	−$200
1	80	100
2	80	100
3	80	100
4	80	

1. **IRR/NPV.** If the opportunity cost of capital is 11%, which of these projects is worth pursuing? (*LO1*)
2. **Mutually Exclusive Investments.** Suppose that you can choose only one of these projects. Which would you choose? The discount rate is still 11%. (*LO5*)
3. **IRR/NPV.** Which project would you choose if the opportunity cost of capital were 16%? (*LO1*)
4. **IRR.** What are the internal rates of return on projects A and B? (*LO2*)
5. **Investment Criteria.** In light of your answers to Quiz Questions 2–4, is there any reason to believe that the project with the higher IRR is the better project? (*LO2*)
6. **Profitability Index.** If the opportunity cost of capital is 11%, what is the profitability index for each project? Does the profitability index rank the projects correctly? (*LO5*)
7. **Payback.** What is the payback period of each project? (*LO3*)
8. **Investment Criteria.** Considering your answers to Quiz Questions 2, 3, and 7, is there any reason to believe that the project with the lower payback period is the better project? (*LO3*)
9. **NPV and IRR.** A project that costs $3,000 to install will provide annual cash flows of $800 for each of the next 6 years. Is this project worth pursuing if the discount rate is 10%? How high can the discount rate be before you would reject the project? (*LO1*)
10. **Payback.** A project that costs $2,500 to install will provide annual cash flows of $600 for the next 6 years. The firm accepts projects with payback periods of less than 5 years. Will the project be accepted? *Should* this project be pursued if the discount rate is 2%? What if the discount rate is 12%? Will the firm's decision change as the discount rate changes? (*LO3*)
11. **Profitability Index.** What is the profitability index of a project that costs $10,000 and provides cash flows of $3,000 in years 1 and 2 and $5,000 in years 3 and 4? The discount rate is 9%. (*LO5*)
12. **NPV.** A proposed nuclear power plant will cost $2.2 billion to build and then will produce cash flows of $300 million a year for 15 years. After that period (in year 15), it must be decommissioned at a cost of $900 million. What is project NPV if the discount rate is 5%? What if it is 18%? (*LO1*)

13. **NPV/IRR.** Consider projects A and B:

Project	Cash Flows (dollars)			NPV at 10%
	C_0	C_1	C_2	
A	−30,000	21,000	21,000	+$6,446
B	−50,000	33,000	33,000	+ 7,273

Calculate IRRs for A and B. Which project does the IRR rule suggest is best? Which project is really best? (*LO2*)

14. **IRR.** You have the chance to participate in a project that produces the following cash flows:

C_0	C_1	C_2
+$5,000	+$4,000	−$11,000

The internal rate of return is 13.6%. If the opportunity cost of capital is 12%, would you accept the offer? (*LO2*)

15. **NPV/IRR.**

a. Calculate the net present value of the following project for discount rates of 0, 50, and 100%: (*LO1*)

C_0	C_1	C_2
−$6,750	+$4,500	+$18,000

b. What is the IRR of the project? (*LO2*)

16. **IRR.** Marielle Machinery Works forecasts the following cash flows on a project under consideration. It uses the internal rate of return rule to accept or reject projects. Should this project be accepted if the required return is 12%? (*LO2*)

C_0	C_1	C_2	C_3
−$10,000	0	+$7,500	+$8,500

17. **NPV/IRR.** A new computer system will require an initial outlay of $20,000, but it will increase the firm's cash flows by $4,000 a year for each of the next 8 years. Is the system worth installing if the required rate of return is 9%? What if it is 14%? How high can the discount rate be before you would reject the project? (*LO1*)

18. **Investment Criteria.** If you insulate your office for $10,000, you will save $1,000 a year in heating expenses. These savings will last forever.

a. What is the NPV of the investment when the cost of capital is 8%? 10%? (*LO1*)
b. What is the IRR of the investment? (*LO2*)
c. What is the payback period on this investment? (*LO3*)

19. **NPV versus IRR.** Here are the cash flows for two mutually exclusive projects:

Project	C_0	C_1	C_2	C_3
A	−$20,000	+$8,000	+$8,000	+$ 8,000
B	− 20,000	0	0	+ 25,000

a. At what interest rates would you prefer project A to B? (*Hint:* Try drawing the NPV profile of each project.). (*LO1*)
b. What is the IRR of each project? (*LO2*)

20. **Payback and NPV.** A project has a life of 10 years and a payback period of 10 years. What must be true of project NPV? (*LO3*)

21. **IRR/NPV.** Consider this project with an internal rate of return of 13.1%. Should you accept or reject the project if the discount rate is 12%? (*LO2*)

Year	Cash Flow
0	+$100
1	−60
2	−60

22. **Payback and NPV.**

a. What is the payback period on each of the following projects? (*LO3*)

	Cash Flows (dollars)				
Project	Year: 0	1	2	3	4
A	−5,000	+1,000	+1,000	+3,000	0
B	−1,000	0	+1,000	+2,000	+3,000
C	−5,000	+1,000	+1,000	+3,000	+5,000

b. Given that you wish to use the payback rule with a cutoff period of 2 years, which projects would you accept? (*LO3*)
c. If you use a cutoff period of 3 years, which projects would you accept? (*LO3*)
d. If the opportunity cost of capital is 10%, which projects have positive NPVs? (*LO1*)
e. "Payback gives too much weight to cash flows that occur after the cutoff date." True or false? (*LO3*)

23. **Profitability Index.** Consider the following projects: (*LO5*)

Project	C_0	C_1	C_2
A	−$2,100	+$2,000	+$1,200
B	− 2,100	+ 1,440	+ 1,728

a. Calculate the profitability index for A and B assuming a 22% opportunity cost of capital.
b. Use the profitability index rule to determine which project(s) you should accept (i) if you could undertake both and (ii) if you could undertake only one.

24. **Capital Rationing.** You are a manager with an investment budget of $8 million. You may invest in the following projects. Investment and cash-flow figures are in millions of dollars. (*LO5*)

Project	Discount Rate, %	Investment	Annual Cash Flow	Project Life, Years
A	10	3	1	5
B	12	4	1	8
C	8	5	2	4
D	8	3	1.5	3
E	12	3	1	6

a. Why might these projects have different discount rates?
b. Which projects should the manager choose?
c. Which projects will be chosen if there is no capital rationing?

25. **Profitability Index versus NPV.** Consider these two projects: (*LO5*)

Project	C_0	C_1	C_2	C_3
A	−\$36	+\$20	+\$20	+\$20
B	− 50	+ 25	+ 25	+ 25

a. Which project has the higher NPV if the discount rate is 10%?
b. Which has the higher profitability index?
c. Which project is most attractive to a firm that can raise an unlimited amount of funds to pay for its investment projects? Which project is most attractive to a firm that is limited in the funds it can raise?

26. **Mutually Exclusive Investments.** Here are the cash flow forecasts for two *mutually exclusive* projects: (*LO1*)

	Cash Flows (dollars)	
Year	Project A	Project B
0	−100	−100
1	30	49
2	50	49
3	70	49

a. Which project would you choose if the opportunity cost of capital is 2%?
b. Which would you choose if the opportunity cost of capital is 12%?
c. Why does your answer change?

27. **Equivalent Annual Annuity.** A precision lathe costs \$10,000 and will cost \$20,000 a year to operate and maintain. If the discount rate is 10% and the lathe will last for 5 years, what is the equivalent annual cost of the tool? (*LO4*)

28. **Equivalent Annual Annuity.** A firm can lease a truck for 4 years at a cost of \$30,000 annually. It can instead buy a truck at a cost of \$80,000, with annual maintenance expenses of \$10,000. The truck will be sold at the end of 4 years for \$20,000. Which is the better option if the discount rate is 10%? (*LO4*)

29. **Multiple IRR.** Consider the following cash flows: (*LO2*)

C_0	C_1	C_2	C_3	C_4
−\$22	+\$20	+\$20	+\$20	−\$40

a. Confirm that one internal rate of return on this project is (a shade above) 7%, and that the other is (a shade below) 34%.
b. Is the project attractive if the discount rate is 5%?
c. What if it is 20%? 40%?
d. Why is the project attractive at midrange discount rates but not at very high or very low rates?

30. **Equivalent Annual Cost.** Econo-Cool air conditioners cost \$300 to purchase, result in electricity bills of \$150 per year, and last for 5 years. Luxury Air models cost \$500, result in electricity bills of \$100 per year, and last for 8 years. The discount rate is 21%. (*LO4*)

a. What are the equivalent annual costs of the Econo-Cool and Luxury Air models?
b. Which model is more cost-effective?
c. Now you remember that the inflation rate is expected to be 10% per year for the foreseeable future. Redo parts (a) and (b).

31. **Investment Timing.** You can purchase an optical scanner today for \$400. The scanner provides benefits worth \$60 a year. The expected life of the scanner is 10 years. Scanners are expected to decrease in price by 20% per year. Suppose the discount rate is 10%. Should you purchase the scanner today or wait to purchase? When is the best purchase time? (*LO4*)

32. **Replacement Decision.** You are operating an old machine that is expected to produce a cash inflow of $5,000 in each of the next 3 years before it fails. You can replace it now with a new machine that costs $20,000 but is much more efficient and will provide a cash flow of $10,000 a year for 4 years. Should you replace your equipment now? The discount rate is 15%. (*LO4*)

33. **Replacement Decision.** A forklift will last for only 2 more years. It costs $5,000 a year to maintain. For $20,000 you can buy a new lift that can last for 10 years and should require maintenance costs of only $2,000 a year. (*LO4*)
    a. If the discount rate is 4% per year, should you replace the forklift?
    b. What if the discount rate is 12% per year? Why does your answer change?

34. **NPV/IRR.** Growth Enterprises believes its latest project, which will cost $80,000 to install, will generate a perpetual growing stream of cash flows. Cash flow at the end of the first year will be $5,000, and cash flows in future years are expected to grow indefinitely at an annual rate of 5%.
    a. If the discount rate for this project is 10%, what is the project NPV? (*LO1*)
    b. What is the project IRR? (*LO2*)

35. **Investment Timing.** A classic problem in management of forests is determining when it is most economically advantageous to cut a tree for lumber. When the tree is young, it grows very rapidly. As it ages, its growth slows down. Why is the NPV-maximizing rule to cut the tree when its growth rate equals the discount rate? (*LO4*)

36. **Multiple IRRs.** Strip Mining Inc. can develop a new mine at an initial cost of $5 million. The mine will provide a cash flow of $30 million in 1 year. The land then must be reclaimed at a cost of $28 million in the second year. (*LO2*)
    a. What are the IRRs of this project?
    b. Should the firm develop the mine if the discount rate is 10%? 20%? 350%? 400%?

37. **Investment Criteria.** A new furnace for your small factory will cost $27,000 a year to install and will require ongoing maintenance expenditures of $1,500 a year. But it is far more fuel-efficient than your old furnace and will reduce your consumption of heating oil by 2,400 gallons per year. Heating oil this year will cost $3 a gallon; the price per gallon is expected to increase by $.50 a year for the next 3 years and then to stabilize for the foreseeable future. The furnace will last for 20 years, at which point it will need to be replaced and will have no salvage value. The discount rate is 8%.
    a. What is the net present value of the investment in the furnace? (*LO1*)
    b. What is the IRR? (*LO2*)
    c. What is the payback period? (*LO3*)
    d. What is the equivalent annual cost of the furnace? (*LO4*)
    e. What is the equivalent annual savings derived from the furnace? (*LO4*)
    f. Compare the PV of the difference between the equivalent annual cost and savings to your answer to part (a). (*LO1*)

Solutions to Chapter 8

Net Present Value and Other Investment Criteria

1. NPV_A = –$200 + [$80 × annuity factor(11%, 4 periods)] =

$$-\$200+\$80\times\left[\frac{1}{0.11}-\frac{1}{0.11\times(1.11)^4}\right]=\$48.20$$

NPV_B = –$200 + [$100 × annuity factor(11%, 3 periods)] =

$$-\$200+\$100\times\left[\frac{1}{0.11}-\frac{1}{0.11\times(1.11)^3}\right]=\$44.37$$

Both projects are worth pursuing.

2. Choose Project A, the project with the higher NPV.

3. NPV_A = –$200 + [$80 × annuity factor(16%, 4 periods)] =

$$-\$200+\$80\times\left[\frac{1}{0.16}-\frac{1}{0.16\times(1.16)^4}\right]=\$23.85$$

NPV_B = –$200 + [$100 × annuity factor(16%, 3 periods)] =

$$-\$200+\$100\times\left[\frac{1}{0.16}-\frac{1}{0.16\times(1.16)^3}\right]=\$24.59$$

Therefore, you should now choose project B.

4. IRR_A = Discount rate (r) which is the solution to the following equation:

$$\$80\times\left[\frac{1}{r}-\frac{1}{r\times(1+r)^4}\right]=\$200\Rightarrow r = IRR_A = 21.86\%$$

IRR_B = Discount rate (r) which is the solution to the following equation:

$$\$100\times\left[\frac{1}{r}-\frac{1}{r\times(1+r)^3}\right]=\$200\Rightarrow r = IRR_B = 23.38\%$$

5. No. Even though project B has the higher IRR, its NPV is lower than that of project A when the discount rate is lower (as in Problem 1) and higher when the discount rate is higher (as in Problem 3). This example shows that the project with the higher IRR is not necessarily better. The IRR of each project is fixed, but as the discount rate increases, project B becomes *relatively* more attractive compared to project A. This is because B's cash flows come earlier, so the present value of these cash flows decreases less rapidly when the discount rate increases.

6. The profitability indexes are as follows:

   Project A: \$48.20/\$200 = 0.2410

   Project B: \$44.37/\$200 = 0.2219

   In this case, *with equal initial investments*, both the profitability index and NPV give projects the same ranking. This is an unusual case, however, since it is rare for the initial investments to be equal.

7. Project A has a payback period of: \$200/\$80 = 2.5 years

   Project B has a payback period of 2 years.

8. No. Despite its longer payback period, Project A may still be the preferred project, for example, when the discount rate is 11% (as in Problems 1 and 2). As in problem 5, you should note that the payback period for each project is fixed, but the NPV changes as the discount rate changes. The project with the shorter payback period need not have the higher NPV.

9. NPV = –\$3,000 + [\$800 × annuity factor(10%, 6 years)] =

$$-\$3{,}000 + \$800 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^6}\right] = \$484.21$$

   At the 10% discount rate, the project is worth pursuing.

   IRR = Discount rate (r) which is the solution to the following equation:

$$\$800 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^6}\right] = \$3{,}000 \Rightarrow r = \text{IRR} = 15.34\%$$

   You can solve for IRR using a financial calculator by entering:
   PV = (–)3000; n = 6; FV = 0; PMT = 800; and then compute i.

   Since the IRR is 15.34%, this is the highest discount rate before project NPV turns negative.

10. Payback period = $2,500/$600 = 4.167 years

This is less than the cutoff, so the firm would accept the project.

r = 2% ⇒ NPV = –$2,500 + [$600 × annuity factor(2%, 6 years)] =

$$-\$2{,}500 + \$600 \times \left[\frac{1}{0.02} - \frac{1}{0.02 \times (1.02)^6}\right] = \$860.86$$

r = 12% ⇒ NPV = –$2,500 + [$600 × annuity factor(12%, 6 years)] =

$$-\$2{,}500 + \$600 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^6}\right] = -\$33.16$$

If r = 2%, the project should be pursued; at r = 12%, it should not be.

11. $$\text{NPV} = -\$10{,}000 + \frac{\$3{,}000}{1.09} + \frac{\$3{,}000}{1.09^2} + \frac{\$5{,}000}{1.09^3} + \frac{\$5{,}000}{1.09^4} = \$2{,}680.38$$

Profitability index = NPV/Investment = 0.2680

12. NPV = –$2.2 billion + [$0.3 billion × annuity factor(r, 15 years)] – [$0.9 billion/(1 + r)^{15}]

$$-\$2.2\,\text{billion} + \$0.3\,\text{billion} \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{15}}\right] + \frac{\$0.9\,\text{billion}}{(1+r)^{15}}$$

r = 5% ⇒ NPV = –$2.2 billion + $2.681 billion = $0.481 billion

r = 18% ⇒ NPV = –$2.2 billion + $1.452 billion = –$0.748 billion

13. IRR_A = Discount rate (r) which is the solution to the following equation:

$$\$21{,}000 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^2}\right] = \$30{,}000 \Rightarrow r = IRR_A = 25.69\%$$

IRR_B = Discount rate (r) which is the solution to the following equation:

$$\$33{,}000 \times \left[\frac{1}{r} + \frac{1}{r \times (1+r)^2}\right] = \$50{,}000 \Rightarrow r = IRR_B = 20.69\%$$

The IRR of project A is 25.69%, and that of B is 20.69%. However, project B has the higher NPV and therefore is preferred. The incremental cash flows of B over A are: –$20,000 at time 0; +$12,000 at times 1 and 2. The NPV of the incremental cash flows (discounted at 10%) is $826.45, which is positive and equal to the difference in the respective project NPVs.

14. $\text{NPV} = \$5{,}000 + \dfrac{\$4{,}000}{1.12} - \dfrac{\$11{,}000}{(1.12)^2} = -\$197.70$

Because the NPV is negative, you should reject the offer. You should reject the offer despite the fact that the IRR exceeds the discount rate. This is a 'borrowing type' project with positive cash flows followed by negative cash flows. A high IRR in these cases is not attractive: You don't want to borrow at a high interest rate.

15. a. r = 0% ⇒ NPV = –\$6,750 + \$4,500 + \$18,000 = \$15,750

$$r = 50\% \Rightarrow \text{NPV} = -\$6{,}750 + \frac{\$4{,}500}{1.50} + \frac{\$18{,}000}{1.50^2} = \$4{,}250$$

$$r = 100\% \Rightarrow \text{NPV} = -\$6{,}750 + \frac{\$4{,}500}{2.00} + \frac{\$18{,}000}{2.00^2} = \$0$$

b. IRR = 100%, the discount rate at which NPV = 0.

16. $\text{NPV} = -\$10{,}000 + \dfrac{\$7{,}500}{1.12^2} + \dfrac{\$8{,}500}{1.12^3} = \$2{,}029.09$

Since the NPV is positive, the project should be accepted.

Alternatively, you can compute the IRR by solving for r, using trial-and-error, in the following equation:

$$-\$10{,}000 + \frac{\$7{,}500}{(1+r)^2} + \frac{\$8{,}500}{(1+r)^3} = 0 \Rightarrow \text{IRR} = 20.61\%$$

Since the IRR of the project is greater than the required rate of return of 12%, the project should be accepted.

17. $\text{NPV}_{9\%}$ = –\$20,000 + [\$4,000 × annuity factor(9%, 8 periods)] =

$$-\$20{,}000 + \$4{,}000 \times \left[\frac{1}{0.09} - \frac{1}{0.09 \times (1.09)^8}\right] = \$2{,}139.28$$

$\text{NPV}_{14\%}$ = –\$20,000 + [\$4,000 × annuity factor(14%, 8 periods)] =

$$-\$20{,}000 + \$4{,}000 \times \left[\frac{1}{0.14} - \frac{1}{0.14 \times (1.14)^8}\right] = -\$1{,}444.54$$

IRR = Discount rate (r) which is the solution to the following equation:

$$\$4{,}000 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^8}\right] = \$20{,}000 \Rightarrow \text{r = IRR = 11.81\%}$$

[Using a financial calculatior, enter: PV = (–)20,000; PMT = 4000; FV = 0; n = 8, and compute i.]

The project will be rejected for any discount rate above this rate.

18. a. The present value of the savings is: $1,000/r

$r = 0.08 \Rightarrow$ PV = \$12,500 and NPV = –\$10,000 + \$12,500 = \$2,500

$r = 0.10 \Rightarrow$ PV = \$10,000 and NPV = –\$10,000 + \$10,000 = \$0

b. IRR = 0.10 = 10%

At this discount rate, NPV = $0

c. Payback period = 10 years

19. a. NPV for each of the two projects, at various discount rates, is tabulated below.

NPV_A = –\$20,000 + [\$8,000 × annuity factor(r%, 3 years)]

$$= -\$20{,}000 + \$8{,}000 \times \left[\frac{1}{r} - \frac{1}{r(1+r)^3}\right]$$

$$NPV_B = -\$20{,}000 + \frac{\$25{,}000}{(1+r)^3}$$

Discount Rate	NPV_A	NPV_B
0%	$4,000	$5,000
2%	3,071	3,558
4%	2,201	2,225
6%	1,384	990
8%	617	-154
10%	-105	-1,217
12%	-785	-2,205
14%	-1,427	-3,126
16%	-2,033	-3,984
18%	-2,606	-4,784
20%	-3,148	-5,532

From the NPV profile, it can be seen that Project A is preferred over Project B if the discount rate is above 4%. At 4% and below, Project B has the higher NPV.

b. IRR_A = Discount rate (r) which is the solution to the following equation:

$$\$8{,}000 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^3}\right] = \$20{,}000 \Rightarrow r = IRR_A = 9.70\%$$

IRR_B = Discount rate (r) which is the solution to the following equation:

$$-\$20{,}000 + \frac{\$25{,}000}{(1+r)^3} = 0 \Rightarrow IRR_B = 7.72\%$$

Using a financial calculator, find IRR_A = 9.70% as follows: enter PV = (–)20; PMT = 8; FV = 0; n = 3; compute i

Find IRR_B = 7.72% as follows: enter PV = (–)20; PMT = 0; FV = 25; n = 3; compute i

20. We know that the *undiscounted* project cash flows sum to the initial investment because payback equals project life. Therefore, the *discounted* cash flows are less than the initial investment, so NPV is negative.

21. $$NPV = \$100 + \frac{-\$60}{1.12} + \frac{-\$60}{(1.12)^2} = -\$1.40$$

IRR_B = Discount rate (r) which is the solution to the following equation:

$$\$100 + \frac{-\$60}{(1+r)} + \frac{-\$60}{(1+r)^2} = 0 \Rightarrow IRR = 13.07\%$$

Because NPV is negative, you should reject the project. This is so despite the fact that the IRR exceeds the discount rate. This is a ‘borrowing type’ project with a positive cash flow followed by negative cash flows. A high IRR in these cases is not attractive: You don’t want to borrow at a high interest rate.

22. a.

Project	Payback
A	3 years
B	2 years
C	3 years

b. Only Project B satisfies the 2-year payback criterion.

c. All three projects satisfy a 3-year payback criterion.

d. $$NPV_A = -\$5{,}000 + \frac{\$1{,}000}{1.10} + \frac{\$1{,}000}{(1.10)^2} + \frac{\$3{,}000}{(1.10)^3} = -\$1{,}010.52$$

$$NPV_B = -\$1{,}000 + \frac{\$1{,}000}{(1.10)^2} + \frac{\$2{,}000}{(1.10)^3} + \frac{\$3{,}000}{(1.10)^4} = \$3{,}378.12$$

$$NPV_C = -\$5{,}000 + \frac{\$1{,}000}{1.10} + \frac{\$1{,}000}{(1.10)^2} + \frac{\$3{,}000}{(1.10)^3} + \frac{\$5{,}000}{(1.10)^4} = \$2{,}405.55$$

e. False. Payback gives *no* weight to cash flows after the cutoff date.

23. a. The net present values of the project cash flows are:

$$NPV_A = -\$2{,}100 + \frac{\$2{,}000}{1.22} + \frac{\$1{,}200}{(1.22)^2} = \$345.58$$

$$NPV_B = -\$2{,}100 + \frac{\$1{,}440}{1.22} + \frac{\$1{,}728}{(1.22)^2} = \$241.31$$

The initial investment for each project is $2,100.

Profitability index (A) = $345.58/$2,100 = 0.1646

Profitability index (B) = $241.31/$2,100 = 0.1149

b. (i) If you could undertake both projects, you should: Both have a positive profitability index.
(ii) If you could undertake only one project, choose A for its higher profitability index.

24. a. The less–risky projects should have lower discount rates.

b. First, find the profitability index of each project.

Project	PV of Cash flow	Investment	NPV	Profitability Index
A	$3.79	$3	$0.79	0.26
B	$4.97	$4	$0.97	0.24
C	$6.62	$5	$1.62	0.32
D	$3.87	$3	$0.87	0.29
E	$4.11	$3	$1.11	0.37

Then, select projects with the highest profitability index until the $8 million budget is exhausted. Therefore, choose Projects E and C.

c. All the projects have positive NPV so that all will be chosen if there is no capital rationing.

25. a. NPV_A = –$36 + [$20 × annuity factor(10%, 3 periods)]

$$= -\$36 + \$20 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^3}\right] = \$13.74$$

NPV_B = –$50 + [$25 × annuity factor(10%, 3 periods)]

$$= -\$50 + \$25 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^3}\right] = \$12.17$$

Thus Project A has the higher NPV if the discount rate is 10%.

b. Project A has the higher profitability index, as shown in the table below:

Project	PV of Cash flow	Investment	NPV	Profitability Index
A	$49.74	$36	$13.74	0.38
B	$62.17	$50	$12.17	0.24

c. A firm with a *limited* amount of funds available should choose Project A since it has a higher profitability index of 0.38, i.e., a higher 'bang for the buck.' Note that A also has a higher NPV as well.

For a firm with unlimited funds, the possibilities are:

i. If the projects are independent projects, then the firm should choose both projects.
ii. However, if the projects are mutually exclusive, then Project A should be selected. It has the higher NPV.

26. a. $NPV_A = -\$100 + \frac{\$30}{1.02} + \frac{\$50}{(1.02)^2} + \frac{\$70}{(1.02)^3} = \$43.43$

NPV_B = – $100 + [$49 × annuity factor(2%, 3 periods)]

$$= -\$100 + \$49 \times \left[\frac{1}{0.02} - \frac{1}{0.02 \times (1.02)^3}\right] = \$41.31$$

If r = 2%, choose A

b. $NPV_A = -\$100 + \frac{\$30}{1.12} + \frac{\$50}{(1.12)^2} + \frac{\$70}{(1.12)^3} = \$16.47$

NPV_B = – \$100 + [\$49 × annuity factor(12%, 3 periods)]

$$= -\$100 + \$49 \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^3}\right] = \$17.69$$

If r = 12%, choose B

c. The larger cash flows of project A tend to come later, so the present value of these cash flows is more sensitive to increases in the discount rate.

27. PV of Costs = \$10,000 + [\$20,000 × annuity factor(10%, 5 years)]

$$= \$10{,}000 + \$20{,}000 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5}\right] = \$85{,}815.74$$

The equivalent annual cost is the payment with the same present value. Solve the following equation for C:

$$C \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^5}\right] = \$85{,}815.74 \Rightarrow C = EAC = \$22{,}637.98$$

Using a financial calculator, enter: n = 5, i = 10, FV = 0; PV = (–)85,815.74; compute PMT

28. Buy: PV of Costs

= \$80,000 + [\$10,000 × annuity factor(10%, 4 years)] – [\$20,000/$(1.10)^4$]

$$= \$80{,}000 + \$10{,}000 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^4}\right] - \frac{\$20{,}000}{(1.10)^4}$$

= \$80,000 + \$31,698.65 – \$13,660.27 = \$98,038.38

The equivalent annual cost is the payment with the same present value. Solve the following equation for C:

$$C \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^4}\right] = \$98{,}038.38 \Rightarrow C = EAC = \$30{,}928.25$$

Using a financial calculator, enter: n = 4, i = 10, FV = 0; PV = (–)98,038.38; compute PMT

If you can lease instead for \$30,000, then this is the less costly option.

You can also compare the PV of the lease costs to the total PV of buying:

$30,000× annuity factor(10%, 4 years) =

$$\$30{,}000\times\left[\frac{1}{0.10}-\frac{1}{0.10\times(1.10)^4}\right]=\$95{,}095.96$$

The PV of the lease costs is less than the PV of the costs when buying the truck.

29. a. The following table shows the NPV profile of the project. NPV is zero at an interest rate between 7% and 8%, and is also equal to zero at an interest rate between 33% and 34%. These are the two IRRs of the project. You can use your calculator to confirm that the two IRRs are, more precisely: 7.16% and 33.67% (as shown below the table).

Discount rate	NPV	Discount rate	NPV
0.00	–2.00	0.21	0.82
0.01	–1.62	0.22	0.79
0.02	–1.28	0.23	0.75
0.03	–0.97	0.24	0.71
0.04	–0.69	0.25	0.66
0.05	–0.44	0.26	0.60
0.06	–0.22	0.27	0.54
0.07	–0.03	0.28	0.47
0.08	0.14	0.29	0.39
0.09	0.29	0.30	0.32
0.10	0.42	0.31	0.24
0.11	0.53	0.32	0.15
0.12	0.62	0.33	0.06
0.13	0.69	0.34	–0.03
0.14	0.75	0.35	–0.13
0.15	0.79	0.36	–0.22
0.16	0.83	0.37	–0.32
0.17	0.85	0.38	–0.42
0.18	0.85	0.39	–0.53
0.19	0.85	0.40	–0.63
0.20	0.84	0.41	–0.74

$$NPV=-\$22+\frac{\$20}{1.0716}+\frac{\$20}{(1.0716)^2}+\frac{\$20}{(1.0716)^3}-\frac{\$40}{(1.0716)^4}=\$0.00$$

$$NPV=-\$22+\frac{\$20}{1.3367}+\frac{\$20}{(1.3367)^2}+\frac{\$20}{(1.3367)^3}-\frac{\$40}{(1.3367)^4}=\$0.00$$

b. At 5% the NPV is:

$$NPV = -\$22 + \frac{\$20}{1.05} + \frac{\$20}{1.05^2} + \frac{\$20}{1.05^3} - \frac{\$40}{1.05^4} = -\$0.443$$

Since the NPV is negative, the project is not attractive.

c. At 20% the NPV is:

$$NPV = -\$22 + \frac{\$20}{1.20} + \frac{\$20}{1.20^2} + \frac{\$20}{1.20^3} - \frac{\$40}{1.20^4} = \$0.840$$

Since the NPV is positive, the project is attractive.

At 40% the NPV is:

$$NPV = -\$22 + \frac{\$20}{1.40} + \frac{\$20}{1.40^2} + \frac{\$20}{1.40^3} - \frac{\$40}{1.40^4} = -\$0.634$$

Since the NPV is negative, the project is not attractive.

d. At a low discount rate, the positive cash flows ($20 for 3 years) are not discounted very much. However, the final cash flow of negative $40 does not get discounted very heavily either. The net effect is a negative NPV.

At very high rates, the positive cash flows are discounted very heavily, resulting in a negative NPV. For moderate discount rates, the positive cash flows that occur in the middle of the project dominate and project NPV is positive.

30. a. Econo-cool costs $300 and lasts for 5 years. The annual rental fee with the same PV is $102.53. We solve as follows:

$$C \times \left[\frac{1}{0.21} - \frac{1}{0.21 \times (1.21)^5}\right] = \$300$$

$C \times$ annuity factor(21%,5 years) = $300

$C \times 2.92598 = \$300 \Rightarrow C = EAC = \$102.53$

The equivalent annual cost of owning *and running* Econo-cool is:

$102.53 + $150 = $252.53

Luxury Air costs $500, and lasts for 8 years. Its equivalent annual rental fee is found as follows:

$$C \times \left[\frac{1}{0.21} - \frac{1}{0.21 \times (1.21)^8}\right] = \$500$$

$C \times$ annuity factor(21%,8 years) = $500

$C \times 3.72558 = \$500 \Rightarrow C = EAC = \$134.21$

The equivalent annual cost of owning and operating Luxury Air is:

$134.21 + $100 = $234.21

b. Luxury Air is more cost effective. It has the lower equivalent annual cost.

c. The real interest rate is now: (1.21/1.10) – 1 = 0.10 = 10%

Redo (a) and (b) using a 10% discount rate. (Note: Because energy costs would normally be expected to inflate along with all other costs, we should assume that the *real* cost of electric bills is either $100 or $150, depending on the model.)

Equivalent annual real cost to own Econo-cool =	$ 79.14
plus $150 (real operating cost) =	150.00
	$229.14

Equivalent annual real cost to own Luxury Air =	$ 93.72
plus $100 (real operating cost) =	100.00
	$193.72

Luxury Air is still more cost effective.

31.

Time until purchase	Cost	NPV at purchase date [a]	NPV today [b]
0	$400.00	–$31.33	–$31.33
1	320.00	48.67	44.25
2	256.00	112.67	93.12
3	204.80	163.87	123.12
4	163.84	204.83	139.90
5	131.07	237.60	147.53
6	104.86	263.81	148.91
7	83.89	284.78	146.14

Notes:

a. – Cost + [60 × annuity factor(10%, 10 years)]

b. (NPV at purchase date)/$(1.10)^n$

NPV is maximized when you wait six years to purchase the scanner.

32. The equivalent annual cost of the new machine is the 4-year annuity with present value equal to $20,000:

$$C \times \left[\frac{1}{0.15} - \frac{1}{0.15 \times (1.15)^4} \right] = \$20{,}000$$

C × annuity factor(15%, 4 years) = $20,000

C × 2.85498 = $20,000 ⇒ C = EAC = $7,005.30

This can be interpreted as the extra yearly charge that should be attributed to the purchase of the new machine spread over its life. It does not yet pay to replace the equipment since the incremental cash flow provided by the new machine is:

$10,000 – $5,000 = $5,000

This is less than the equivalent annual cost of the new machine.

33. a. The equivalent annual cost (EAC) of the *new* machine over its 10-year life is found by solving as follows:

$$C \times \left[\frac{1}{0.04} - \frac{1}{0.04 \times (1.04)^{10}} \right] = \$20{,}000$$

C × annuity factor(4%, 10 years) = $20,000

C × 8.11090 = $20,000 ⇒ C = EAC = $2,465.82

Together with maintenance costs of $2,000 per year, the equivalent cost of owning *and* operating is: $4,465.82

The old machine costs $5,000 per year to operate, and is already paid for. (We assume it has no scrap value and therefore no opportunity cost.) The new machine is less costly. You should replace.

b. If r = 12%, then the equivalent annual cost (EAC) is computed as follows:

$$C \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^{10}} \right] = \$20{,}000$$

C × annuity factor(12%, 10 years) = $20,000

C × 5.65022 = $20,000 ⇒ C = EAC = $3,539.68

The equivalent cost of owning and operating the new machine is now: $5,539.68 This is higher than that of the old machine. Do not replace.

Your answer changes because the higher discount rate implies that the opportunity cost of the money tied up in the forklift also is higher.

34. a. Present Value = $\dfrac{\text{Cash flow at end of year}}{\text{Discount rate - growth rate}} = \dfrac{\$5{,}000}{0.10 - 0.05} = \$100{,}000$

NPV = –$80,000 + $100,000 = $20,000

b. Recall that the IRR is the discount rate that makes NPV equal to zero:

(– Investment) + (PV of cash flows discounted at IRR) = 0

$$-\$80{,}000 + \frac{\$5{,}000}{\text{IRR} - 0.05} = 0$$

Solving, we find that:

IRR = ($5,000/$80,000) + 0.05 = 0.1125 = 11.25%

35. For harvesting lumber, the NPV-maximizing rule is to cut the tree when its growth rate equals the discount rate. When the tree is young and the growth rate exceeds the discount rate, it pays to wait: the value of the tree is increasing at a rate that exceeds the discount rate. When the tree is older and the growth rate is less than r, cutting immediately is better, since the revenue from the tree can be invested to earn the rate r, which is greater than the growth rate the tree is providing.

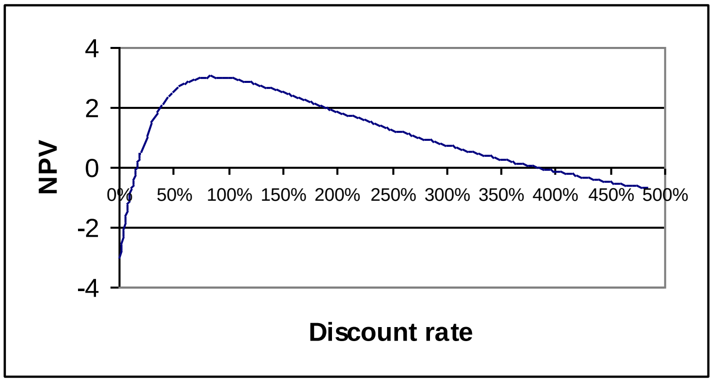
36. a.

Time	Cash flow
0	–$5 million
1	30 million
2	–28 million

The graph below shows a plot of NPV as a function of the discount rate. NPV = 0 when r equals (approximately) either 15.61% or 384.39%. These are the two IRRs.

b.

Discount rate	NPV	Develop?
10%	–$0.868 million	No
20%	0.556	Yes
350%	0.284	Yes
400%	–0.120	No



37.

a.

$$NPV = -\$27{,}000 + \frac{(-1500 + (2400 \times \$3.50)}{1.08} + \frac{(-1500 + (2400 \times \$4.00)}{1.08^2}$$
$$+ \frac{(-1500 + (2400 \times \$4.50)}{1.08^3} + \frac{(-1500 + (2400 \times \$4.50)}{1.08^4} + \ldots + \frac{(-1500 + (2400 \times \$4.50)}{1.08^{20}} = \$61{,}058$$

b. Using Excel, IRR = 31.37%

c. Cumulative Cash Flows are positive after year 4.

Year	CF	CUM CF
0	-27000	-27000
1	6900	-20100
2	8100	-12000
3	9300	-2700
4	9300	6600

d. The equivalent annual cost of the new machine is the 20-year annuity with present value equal to $27,000:

$$C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}}\right] = \$27{,}000$$

C × annuity factor(8%, 20 years) = $270,000

$$C \times 9.8181 = \$27{,}000 \Rightarrow C = EAC = \$2{,}750.02$$

e. The present value of the annual savings is given by the following equation:

$$PV = \frac{(-1500 + (2400 \times \$3.50)}{1.08} + \frac{(-1500 + (2400 \times \$4.00)}{1.08^2}$$
$$+ \frac{(-1500 + (2400 \times \$4.50)}{1.08^3} + ... + \frac{(-1500 + (2400 \times \$4.50)}{1.08^{20}} = \$88{,}058$$

The equivalent annual annuity for this present value at 8% for 20 years is $8,968.92.

EAA = Present Value of Annual Savings / annuity factor(8%, 20 years)
= $88,058/9.8181 = $8,968.92

The difference between equivalent annual savings and costs is $6,219 ($8,969 - $2,750). This value is equivalent to an annual annuity with a present value of $61,058, the net present value from part a.

$6,219 x annuity factor(8%, 20 years) = $6,219 x 9.8181 = $61,058.

Solution to Minicase for Chapter 7

None of the measures in the summary tables is appropriate for the analysis of this case, although the NPV calculations can be used as the starting point for an appropriate analysis.

The payback period is not appropriate for the same reasons that it is always inappropriate for analysis of a capital budgeting problem: cash flows after the payback period are ignored; cash flows before the payback period are all assigned equal weight, regardless of timing; the cutoff period is arbitrary.

The internal rate of return criterion can result in incorrect rankings among mutually exclusive investment projects when there are differences in the size of the projects under consideration and/or when there are differences in the timing of the cash flows. In choosing between the two different stamping machines, both of these differences exist.

The net present value calculations indicate that the Skilboro machines have a greater NPV ($2.56 million) than do the Munster machines ($2.40 million). However, since the Munster machines also have a shorter life, it is not clear whether the difference in NPV is simply a matter of longevity. In order to adjust for this difference, we can compute the equivalent annual annuity for each:

Munster machines:

$$C \times \left[\frac{1}{0.15} - \frac{1}{0.15 \times (1.15)^7}\right] = \$2.40 \text{ million}$$

C × annuity factor(15%, 7 years) = $2.40 million

C × 4.16042 = $2.40 million ⇒ C = EAC = $0.57686 million

Skilboro machines:

$$C \times \left[\frac{1}{0.15} - \frac{1}{0.15 \times (1.15)^{10}}\right] = \$2.56 \text{ million}$$

C × annuity factor(15%, 10 years) = $2.56 million

C × 5.01877 = $2.56 million ⇒ C = EAC = $0.51009 million

Therefore, the Munster machines are preferred.

Another approach to making this comparison is to compute the equivalent annual annuity based on the cost of the two machines. The cost of the Munster machine is $8 million, so that the equivalent annual annuity is computed as follows:

Munster machines:

$$C \times \left[\frac{1}{0.15} - \frac{1}{0.15 \times (1.15)^7} \right] = \$8 \text{ million}$$

C × annuity factor(15%, 7 years) = $8 million

C × 4.16042 = $8 million ⇒ C = EAC = $1.92288 million

For the Skilboro machine, we can treat the reduction in operator and material cost as a reduction in the present value of the cost of the machine:

$$PV = \$500{,}000 \times \left[\frac{1}{0.15} - \frac{1}{0.15 \times (1.15)^{10}} \right] = \$2.50938 \text{ million}$$

$12.5 million – $2.50938 million = $9.99062 million

$$C \times \left[\frac{1}{0.15} - \frac{1}{0.15 \times (1.15)^{10}} \right] = \$9.99062 \text{ million}$$

C × annuity factor(15%, 10 years) = $9.99062 million

C × 5.01877 = $9.99062 million ⇒ C = EAC = $1.99065 million

Here, the equivalent annual cost is less for the Munster machines.

Note that the differences in the equivalent annual annuities for the two methods are equal. (Differences are due to rounding.)