1. **Present Values.** Compute the present value of a \$100 cash flow for the following combinations of discount rates and times: (*LO2*)
   a. $r = 8\%$, $t = 10$ years.
   b. $r = 8\%$, $t = 20$ years.
   c. $r = 4\%$, $t = 10$ years.
   d. $r = 4\%$, $t = 20$ years.
2. **Future Values.** Compute the future value of a \$100 cash flow for the same combinations of rates and times as in Quiz Question 1. (*LO1*)
3. **Future Values.** In 1880 five aboriginal trackers were each promised the equivalent of 100 Australian dollars for helping to capture the notorious outlaw Ned Kelley. In 1993 the granddaughters of two of the trackers claimed that this reward had not been paid. The Victorian prime minister stated that if this was true, the government would be happy to pay the \$100. However, the granddaughters also claimed that they were entitled to compound interest. How much was each entitled to if the interest rate was 4%? What if it was 8%? (*LO1*)
4. **Future Values.** You deposit \$1,000 in your bank account. If the bank pays 4% simple interest, how much will you accumulate in your account after 10 years? What if the bank pays compound interest? How much of your earnings will be interest on interest? (*LO1*)
5. **Present Values.** You will require \$700 in 5 years. If you earn 5% interest on your funds, how much will you need to invest today in order to reach your savings goal? (*LO2*)
6. **Calculating Interest Rate.** Find the interest rate implied by the following combinations of present and future values: (*LO4*)

Present Value	Years	Future Value
\$400	11	\$684
183	4	249
300	7	300

7. **Present Values.** Would you rather receive \$1,000 a year for 10 years or \$800 a year for 15 years if
   a. the interest rate is 5%? (*LO3*)
   b. the interest rate is 20%? (*LO3*)

   Why do your answers to (a) and (b) differ?

8. **Calculating Interest Rate.** Find the annual interest rate. (*LO4*)

Present Value	Future Value	Time Period
$100	$115.76	3 years
200	262.16	4
100	110.41	5

9. **Present Values.** What is the present value of the following cash-flow stream if the interest rate is 6%? (*LO3*)

Year	Cash Flow
1	$200
2	400
3	300

10. **Number of Periods.** How long will it take for $400 to grow to $1,000 at the interest rate specified? (*LO1*)
    a. 4%
    b. 8%
    c. 16%

11. **Calculating Interest Rate.** Find the effective annual interest rate for each case. (*LO5*)

APR	Compounding Period
12%	1 month
8	3
10	6

12. **Calculating Interest Rate.** Find the APR (the stated interest rate) for each case. (*LO5*)

Effective Annual Interest Rate	Compounding Period
10.00%	1 month
6.09	6
8.24	3

13. **Growth of Funds.** If you earn 6% per year on your bank account, how long will it take an account with $100 to double to $200? (*LO1*)

14. **Comparing Interest Rates.** Suppose you can borrow money at 8.6% per year (APR) compounded semiannually or 8.4% per year (APR) compounded monthly. Which is the better deal? (*LO5*)

15. **Calculating Interest Rate.** Lenny Loanshark charges "1 point" per week (that is, 1% per week) on his loans. What APR must he report to consumers? Assume exactly 52 weeks in a year. What is the effective annual rate? (*LO5*)

16. **Compound Interest.** Investments in the stock market have increased at an average compound rate of about 5% since 1900. It is now 2007.
    a. If you invested $1,000 in the stock market in 1900, how much would that investment be worth today? (*LO1*)
    b. If your investment in 1900 has grown to $1 million, how much did you invest in 1900? (*LO2*)

17. **Compound Interest.** Old Time Savings Bank pays 4% interest on its savings accounts. If you deposit $1,000 in the bank and leave it there, how much interest will you earn in the first year? The second year? The tenth year? (*LO1*)

18. **Compound Interest.** New Savings Bank pays 4% interest on its deposits. If you deposit $1,000 in the bank and leave it there, will it take more or less than 25 years for your money to double? You should be able to answer this without a calculator or interest rate tables. (*LO1*)

19. **Calculating Interest Rate.** A zero-coupon bond that will pay $1,000 in 10 years is selling today for $422.41. What interest rate does the bond offer? (*LO4*)

20. **Present Values.** A famous quarterback just signed a $15 million contract providing $3 million a year for 5 years. A less famous receiver signed a $14 million 5-year contract providing $4 million now and $2 million a year for 5 years. Who is better paid? The interest rate is 10%. (*LO3*)

21. **Compound Growth.** In September 2007 a pound of apples cost $1.18, while oranges cost $1.50. Ten years earlier the price of apples was only $.93 a pound and that of oranges was $.96 a pound. What was the annual compound rate of growth in the price of the two fruits? If the same rates of growth persist in the future, what will be the price of apples in 2027? What about the price of oranges? (*LO6*)

22. **Loan Payments.** If you take out an $8,000 car loan that calls for 48 monthly payments at an APR of 10%, what is your monthly payment? What is the effective annual interest rate on the loan? (*LO5*)

23. **Annuity Values.** (*LO3*)
    a. What is the present value of a 3-year annuity of $100 if the discount rate is 6%?
    b. What is the present value of the annuity in (a) if you have to wait 2 years instead of 1 year for the first payment?

24. **Annuities and Interest Rates.** Professor's Annuity Corp. offers a lifetime annuity to retiring professors. For a payment of $80,000 at age 65, the firm will pay the retiring professor $600 a month until death.
    a. If the professor's remaining life expectancy is 20 years, what is the monthly rate on this annuity? What is the effective annual rate? (*LO4*)
    b. If the monthly interest rate is .5%, what monthly annuity payment can the firm offer to the retiring professor? (*LO3*)

25. **Annuity Values.** You want to buy a new car, but you can make an initial payment of only $2,000 and can afford monthly payments of at most $400. (*LO3*)
    a. If the APR on auto loans is 12% and you finance the purchase over 48 months, what is the maximum price you can pay for the car?
    b. How much can you afford if you finance the purchase over 60 months?

26. **Calculating Interest Rate.** In a *discount interest loan,* you pay the interest payment up front. For example, if a 1-year loan is stated as \$10,000 and the interest rate is 10%, the borrower "pays" $.10 \times \$10{,}000 = \$1{,}000$ immediately, thereby receiving net funds of \$9,000 and repaying \$10,000 in a year. (*LO5*)
    a. What is the effective interest rate on this loan?
    b. If you call the discount d (for example, $d = 10\%$ using our numbers), express the effective annual rate on the loan as a function of d.
    c. Why is the effective annual rate always greater than the stated rate d?

27. **Annuity Due.** Recall that an annuity due is like an ordinary annuity except that the first payment is made immediately instead of at the end of the first period. (*LO3*)
    a. Why is the present value of an annuity due equal to $(1 + r)$ times the present value of an ordinary annuity?
    b. Why is the future value of an annuity due equal to $(1 + r)$ times the future value of an ordinary annuity?

28. **Rate on a Loan.** If you take out an \$8,000 car loan that calls for 48 monthly payments of \$240 each, what is the APR of the loan? What is the effective annual interest rate on the loan? (*LO5*)

29. **Loan Payments.** Reconsider the car loan in the previous question. What if the payments are made in four annual year-end installments? What annual payment would have the same present value as the monthly payment you calculated? Use the same effective annual interest rate as in the previous question. Why is your answer not simply 12 times the monthly payment? (*LO5*)

30. **Annuity Value.** Your landscaping company can lease a truck for $8,000 a year (paid at year-end) for 6 years. It can instead buy the truck for $40,000. The truck will be valueless after 6 years. If the interest rate your company can earn on its funds is 7%, is it cheaper to buy or lease? (*LO3*)

31. **Annuity-Due Value.** Reconsider the previous problem. What if the lease payments are an annuity due, so that the first payment comes immediately? Is it cheaper to buy or lease? (*LO3*)

32. **Annuity Due.** A store offers two payment plans. Under the installment plan, you pay 25% down and 25% of the purchase price in each of the next 3 years. If you pay the entire bill immediately, you can take a 10% discount from the purchase price. Which is a better deal if you can borrow or lend funds at a 5% interest rate? (*LO3*)

33. **Annuity Value.** Reconsider the previous question. How will your answer change if the payments on the 4-year installment plan do not start for a full year? (*LO3*)

34. **Annuity and Annuity-Due Payments.** (*LO3*)
    a. If you borrow $1,000 and agree to repay the loan in five equal annual payments at an interest rate of 12%, what will your payment be?
    b. What if you make the first payment on the loan immediately instead of at the end of the first year?

35. **Valuing Delayed Annuities.** Suppose that you will receive annual payments of $10,000 for a period of 10 years. The first payment will be made 4 years from now. If the interest rate is 5%, what is the present value of this stream of payments? (*LO3*)

36. **Mortgage with Points.** Home loans typically involve "points," which are fees charged by the lender. Each point charged means that the borrower must pay 1% of the loan amount as a fee. For example, if the loan is for $100,000 and 2 points are charged, the loan repayment schedule is calculated on a $100,000 loan but the net amount the borrower receives is only $98,000. What is the effective annual interest rate charged on such a loan assuming loan repayment occurs over 360 months? Assume the interest rate is 1% per month. (*LO3*)

37. **Amortizing Loan.** You take out a 30-year $100,000 mortgage loan with an APR of 6% and monthly payments. In 12 years you decide to sell your house and pay off the mortgage. What is the principal balance on the loan? (*LO3*)

38. **Amortizing Loan.** Consider a 4-year amortizing loan. You borrow $1,000 initially, and repay it in four equal annual year-end payments. (*LO3*)

a. If the interest rate is 8%, show that the annual payment is $301.92.

b. Fill in the following table, which shows how much of each payment is interest versus principal repayment (that is, amortization), and the outstanding balance on the loan at each date.

Time	Loan Balance	Year-End Interest Due on Balance	Year-End Payment	Amortization of Loan
0	$1,000	$80	$301.92	$221.92
1	———	———	301.92	———
2	———	———	301.92	———
3	———	———	301.92	———
4	0	0	—	—

c. Show that the loan balance after 1 year is equal to the year-end payment of $301.92 times the 3-year annuity factor.

39. **Annuity Value.** You've borrowed $4,248.68 and agreed to pay back the loan with monthly payments of $200. If the interest rate is 12% stated as an APR, how long will it take you to pay back the loan? What is the effective annual rate on the loan? (*LO3*)

40. **Annuity Value.** The $40 million lottery payment that you just won actually pays $2 million per year for 20 years. If the discount rate is 8% and the first payment comes in 1 year, what is the present value of the winnings? What if the first payment comes immediately? (*LO3*)

41. **Real Annuities.** A retiree wants level consumption in real terms over a 30-year retirement. If the inflation rate equals the interest rate she earns on her $450,000 of savings, how much can she spend in real terms each year over the rest of her life? (*LO6*)

42. **EAR versus APR.** You invest $1,000 at a 6% annual interest rate, stated as an APR. Interest is compounded monthly. How much will you have in 1 year? In 1.5 years? (*LO5*)

43. **Annuity Value.** You just borrowed $100,000 to buy a condo. You will repay the loan in equal monthly payments of $804.62 over the next 30 years. What monthly interest rate are you paying on the loan? What is the effective annual rate on that loan? What rate is the lender more likely to quote on the loan? (*LO3*)

44. **EAR.** If a bank pays 6% interest with continuous compounding, what is the effective annual rate? (*LO5*)

45. **Annuity Values.** You can buy a car that is advertised for $24,000 on the following terms: (a) pay $24,000 and receive a $2,000 rebate from the manufacturer; (b) pay $500 a month for 4 years for total payments of $24,000, implying zero percent financing. Which is the better deal if the interest rate is 1% per month? (*LO3*)

46. **Continuous Compounding.** How much will $100 grow to if invested at a continuously compounded interest rate of 10% for 8 years? What if it is invested for 10 years at 8%? (*LO5*)

47. **Future Values.** I now have $20,000 in the bank earning interest of .5% per month. I need $30,000 to make a down payment on a house. I can save an additional $100 per month. How long will it take me to accumulate the $30,000? (*LO3*)

48. **Perpetuities.** A local bank advertises the following deal: "Pay us $100 a year for 10 years and then we will pay you (or your beneficiaries) $100 a year *forever.*" Is this a good deal if the interest rate available on other deposits is 6%? (*LO3*)

49. **Perpetuities.** A local bank will pay you $100 a year for your lifetime if you deposit $2,500 in the bank today. If you plan to live forever, what interest rate is the bank paying? (*LO4*)

50. **Perpetuities.** A property will provide $10,000 a year forever. If its value is $125,000, what must be the discount rate? (*LO4*)

51. **Applying Time Value.** You can buy property today for \$3 million and sell it in 5 years for \$4 million. (You earn no rental income on the property.) (*LO3*)

   a. If the interest rate is 8%, what is the present value of the sales price?
   b. Is the property investment attractive to you? Why or why not?
   c. Would your answer to (b) change if you also could earn \$200,000 per year rent on the property?

52. **Applying Time Value.** A factory costs \$400,000. You forecast that it will produce cash inflows of \$120,000 in year 1, \$180,000 in year 2, and \$300,000 in year 3. The discount rate is 12%. Is the factory a good investment? Explain. (*LO3*)

53. **Applying Time Value.** You invest \$1,000 today and expect to sell your investment for \$2,000 in 10 years. (*LO1*)

   a. Is this a good deal if the discount rate is 6%?
   b. What if the discount rate is 10%?

54. **Calculating Interest Rate.** A store will give you a 3% discount on the cost of your purchase if you pay cash today. Otherwise, you will be billed the full price with payment due in 1 month. What is the implicit borrowing rate being paid by customers who choose to defer payment for the month? (*LO4*)

55. **Quoting Rates.** Banks sometimes quote interest rates in the form of "add-on interest." In this case, if a 1-year loan is quoted with a 20% interest rate and you borrow \$1,000, then you pay back \$1,200. But you make these payments in monthly installments of \$100 each. What are the true APR and effective annual rate on this loan? Why should you have known that the true rates must be greater than 20% even before doing any calculations? (*LO5*)

56. **Compound Interest.** Suppose you take out a \$1,000, 3-year loan using add-on interest (see previous problem) with a quoted interest rate of 20% per year. What will your monthly payments be? (Total payments are $\$1,000 + \$1,000 \times .20 \times 3 = \$1,600$.) What are the true APR and effective annual rate on this loan? Are they the same as in the previous problem? (*LO5*)

57. **Calculating Interest Rate.** What is the effective annual rate on a 1-year loan with an interest rate quoted on a discount basis (see Practice Problem 26) of 20%? (*LO4*)

58. **Effective Rates.** First National Bank pays 6.2% interest compounded semiannually. Second National Bank pays 6% interest, compounded monthly. Which bank offers the higher effective annual rate? (*LO5*)

59. **Calculating Interest Rate.** You borrow $1,000 from the bank and agree to repay the loan over the next year in 12 equal monthly payments of $90. However, the bank also charges you a loan-initiation fee of $20, which is taken out of the initial proceeds of the loan. What is the effective annual interest rate on the loan taking account of the impact of the initiation fee? (*LO4*)

60. **Retirement Savings.** You believe you will need to have saved $500,000 by the time you retire in 40 years in order to live comfortably. If the interest rate is 6% per year, how much must you save each year to meet your retirement goal? (*LO3*)

61. **Retirement Savings.** How much would you need in the previous problem if you believe that you will inherit $100,000 in 10 years? (*LO3*)

62. **Retirement Savings.** You believe you will spend $40,000 a year for 20 years once you retire in 40 years. If the interest rate is 6% per year, how much must you save each year until retirement to meet your retirement goal? (*LO3*)

63. **Retirement Planning.** A couple thinking about retirement decide to put aside $3,000 each year in a savings plan that earns 8% interest. In 5 years they will receive a gift of $10,000 that also can be invested. (*LO3*)
    a. How much money will they have accumulated 30 years from now?
    b. If their goal is to retire with $800,000 of savings, how much extra do they need to save every year?

64. **Retirement Planning.** A couple will retire in 50 years; they plan to spend about $30,000 a year in retirement, which should last about 25 years. They believe that they can earn 8% interest on retirement savings. (*LO3*)

   a. If they make annual payments into a savings plan, how much will they need to save each year? Assume the first payment comes in 1 year.
   b. How would the answer to part (a) change if the couple also realize that in 20 years, they will need to spend $60,000 on their child's college education?

65. **Real versus Nominal Dollars.** An engineer in 1950 was earning $6,000 a year. Today she earns $60,000 a year. However, on average, goods today cost 6.9 times what they did in 1950. What is her real income today in terms of constant 1950 dollars? (*LO6*)

66. **Real versus Nominal Rates.** If investors are to earn a 3% real interest rate, what nominal interest rate must they earn if the inflation rate is

   a. zero? (*LO6*)
   b. 4%? (*LO6*)
   c. 6%? (*LO6*)

67. **Real Rates.** If investors receive a 6% interest rate on their bank deposits, what real interest rate will they earn if the inflation rate over the year is

   a. zero? (*LO6*)
   b. 3%? (*LO6*)
   c. 6%? (*LO6*)

68. **Real versus Nominal Rates.** You will receive $100 from a savings bond in 3 years. The nominal interest rate is 8%.

   a. What is the present value of the proceeds from the bond? (*LO2*)
   b. If the inflation rate over the next few years is expected to be 3%, what will the real value of the $100 payoff be in terms of today's dollars? (*LO6*)
   c. What is the real interest rate? (*LO6*)
   d. Show that the real payoff from the bond [from part (b)] discounted at the real interest rate [from part (c)] gives the same present value for the bond as you found in part (a). (*LO6*)

69. **Real versus Nominal Dollars.** Your consulting firm will produce cash flows of $100,000 this year, and you expect cash flow to keep pace with any increase in the general level of prices. The interest rate currently is 6%, and you anticipate inflation of about 2%.

   a. What is the present value of your firm's cash flows for years 1 through 5? (*LO6*)
   b. How would your answer to (a) change if you anticipated no growth in cash flow? (*LO2*)

70. **Real versus Nominal Annuities.** Good news: You will almost certainly be a millionaire by the time you retire in 50 years. Bad news: The inflation rate over your lifetime will average about 3%. (*LO6*)

   a. What will be the real value of $1 million by the time you retire in terms of today's dollars?
   b. What real annuity (in today's dollars) will $1 million support if the real interest rate at retirement is 2% and the annuity must last for 20 years?

71. **Real versus Nominal.** If the interest rate is 6% per year, how long will it take for your money to *quadruple* in value? If the inflation rate is 4% per year, what will be the change in the purchasing power of your money over this period? (*LO1, 6*)

72. **Inflation.** In the summer of 2007, Zimbabwe's official inflation rate was about 110% per month. What was the annual inflation rate? (*LO6*)

73. **Perpetuities.** British government 4% perpetuities pay £4 interest each year forever. Another bond, 2½% perpetuities, pays £2.50 a year forever. What is the value of 4% perpetuities if the long-term interest rate is 6%? What is the value of 2½% perpetuities? (*LO3*)

74. **Real versus Nominal Annuities.** (*LO6*)

   a. You plan to retire in 30 years and want to accumulate enough by then to provide yourself with $30,000 a year for 15 years. If the interest rate is 10%, how much must you accumulate by the time you retire?
   b. How much must you save each year until retirement in order to finance your retirement consumption?
   c. Now you remember that the annual inflation rate is 4%. If a loaf of bread costs $1 today, what will it cost by the time you retire?
   d. You really want to consume $30,000 a year in *real* dollars during retirement and wish to save an equal *real* amount each year until then. What is the real amount of savings that you need to accumulate by the time you retire?
   e. Calculate the required preretirement real annual savings necessary to meet your consumption goals. Compare to your answer to (b). Why is there a difference?
   f. What is the nominal value of the amount you need to save during the first year? (Assume the savings are put aside at the end of each year.) The thirtieth year?

75. **Retirement and Inflation.** Redo part (a) of Practice Problem 64, but now assume that the inflation rate over the next 50 years will average 4%. (*LO6*)

   a. What is the real annual savings the couple must set aside?
   b. How much do they need to save in nominal terms in the first year?
   c. How much do they need to save in nominal terms in the last year?
   d. What will be their nominal expenditures in the first year of retirement? The last?

76. **Perpetuities.** What is the value of a perpetuity that pays $100 every 3 months forever? The discount rate quoted on an APR basis is 6%. (*LO5*)

77. **Changing Interest Rates.** If the interest rate this year is 8% and the interest rate next year will be 10%, what is the future value of $1 after 2 years? What is the present value of a payment of $1 to be received in 2 years? (*LO1, 2*)

78. **Changing Interest Rates.** Your wealthy uncle established a $1,000 bank account for you when you were born. For the first 8 years of your life, the interest rate earned on the account was 6%. Since then, rates have been only 4%. Now you are 21 years old and ready to cash in. How much is in your account? (*LO1*)

79. **Real versus Nominal Cash Flows.**

   a. It is 2010, you've just graduated college, and you are contemplating your lifetime budget. You think your general living expenses will average around $50,000 a year. For the next 8 years, you will rent an apartment for $16,000 a year. After that, you will want to buy a house that should cost around $250,000. In addition, you will need to buy a new car roughly once every 10 years, costing around $30,000 each. In 25 years, you will have to put aside around

$150,000 to put a child through college, and in 30 years you'll need to do the same for another child. In 50 years, you will retire, and will need to have accumulated enough savings to support roughly 20 years of retirement spending of around $35,000 a year on top of your social security benefits. The interest rate is 5% per year. What average salary will you need to earn to support this lifetime consumption plan? (*LO3*)

b. Whoops! You just realized that the inflation rate over your lifetime is likely to average about 3% per year, and you need to redo your calculations. As a rough cut, it seems reasonable to assume that all relevant prices and wages will increase at around the rate of inflation. What is your new estimate of the required salary (in today's dollars)? (*LO6*)

80. **Amortizing Loans and Inflation.** Suppose you take out a $100,000, 20-year mortgage loan to buy a condo. The interest rate on the loan is 6%, and to keep things simple, we will assume you make payments on the loan annually at the end of each year. (*LO3*)

a. What is your annual payment on the loan?

b. Construct a mortgage amortization table in Excel similar to Table 5–5 in which you compute the interest payment each year, the amortization of the loan, and the loan balance each year. (Allow the interest rate to be an input that the user of the spreadsheet can enter and change.)

c. What fraction of your initial loan payment is interest? What fraction is amortization? What about the last loan payment? What fraction of the loan has been paid off after 10 years (halfway through the life of the loan)?

d. If the inflation rate is 2%, what is the real value of the first (year-end) payment? The last?

e. Now assume the inflation rate is 8% and the real interest rate on the loan is unchanged. What must be the new nominal interest rate? Recompute the amortization table. What is the real value of the first (year-end) payment in this high-inflation scenario? The real value of the last payment?

f. Comparing your answers to (d) and (e), can you see why high inflation rates might hurt the real estate market?

Solutions to Chapter 5

The Time Value of Money

1. a. $\$100/(1.08)^{10} = \$46.32$

   b. $\$100/(1.08)^{20} = \$21.45$

   c. $\$100/(1.04)^{10} = \$67.56$

   d. $\$100/(1.04)^{20} = \$45.64$

2. a. $\$100 \times (1.08)^{10} = \$215.89$

   b. $\$100 \times (1.08)^{20} = \$466.10$

   c. $\$100 \times (1.04)^{10} = \$148.02$

   d. $\$100 \times (1.04)^{20} = \$219.11$

3. $\$100 \times (1.04)^{113} = \$8,409.45$

   $\$100 \times (1.08)^{113} = \$598,252.29$

4. With simple interest, you earn 4% of $1,000 or $40 each year. There is no interest on interest. After 10 years, you earn total interest of $400, and your account accumulates to $1,400. With compound interest, your account grows to: $\$1,000 \times (1.04)^{10} = \$1480.24$ Therefore $80.24 is interest on interest.

5. $PV = \$700/(1.05)^5 = \$548.47$

6.

	Present Value	Years	Future Value	Interest Rate
a.	\$400	11	\$684	$\left[\frac{684}{400}\right]^{(1/11)} - 1 = 5.00\%$
b.	\$183	4	\$249	$\left[\frac{249}{183}\right]^{(1/4)} - 1 = 8.00\%$
c.	\$300	7	\$300	$\left[\frac{300}{300}\right]^{(1/7)} - 1 = 0\%$

To find the interest rate, we rearrange the basic future value equation as follows:

$$\text{FV} = \text{PV} \times (1 + r)^t \Rightarrow r = \left[\frac{\text{FV}}{\text{PV}}\right]^{(1/t)} - 1$$

7. You should compare the present values of the two annuities.

a.

$$\text{PV} = \$1{,}000 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{10}}\right] = \$7{,}721.73$$

$$\text{PV} = \$800 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{15}}\right] = \$8{,}303.73$$

b.

$$\text{PV} = \$1{,}000 \times \left[\frac{1}{0.20} - \frac{1}{0.20 \times (1.20)^{10}}\right] = \$4{,}192.47$$

$$\text{PV} = \$800 \times \left[\frac{1}{0.20} - \frac{1}{0.20 \times (1.20)^{15}}\right] = \$3{,}740.38$$

c. When the interest rate is low, as in part (a), the longer (i.e., 15-year) but smaller annuity is more valuable because the impact of discounting on the present value of future payments is less significant.

8. $\$100 \times (1 + r)^3 = \$115.76 \Rightarrow r = 5.00\%$

$\$200 \times (1 + r)^4 = \$262.16 \Rightarrow r = 7.00\%$

$\$100 \times (1 + r)^5 = \$110.41 \Rightarrow r = 2.00\%$

9. $PV = (\$200/1.06) + (\$400/1.06^2) + (\$300/1.06^3) = \$188.68 + \$356.00 + \$251.89 = \$796.57$

10. In these problems, you can either solve the equation provided directly, or you can use your financial calculator, setting: PV = (–)400, FV = 1000, PMT = 0, i as specified by the problem. Then compute n on the calculator.

    a. $\$400 \times (1.04)^t = \$1,000 \Rightarrow t = 23.36$ periods

    b. $\$400 \times (1.08)^t = \$1,000 \Rightarrow t = 11.91$ periods

    c. $\$400 \times (1.16)^t = \$1,000 \Rightarrow t = 6.17$ periods

11.

	APR	Compounding period	Effective annual rate
a.	12%	1 month (m = 12/yr)	$1.01^{12} - 1 = 0.1268 = 12.68\%$
b.	8%	3 months (m = 4/yr)	$1.02^4 - 1 = 0.0824 = 8.24\%$
c.	10%	6 months (m = 2/yr)	$1.05^2 - 1 = 0.1025 = 10.25\%$

12.

	Effective Rate	Compounding period	Per period rate	APR
a.	10.00%	1 month (m = 12/yr)	$1.10^{(1/12)} - 1 = 0.0080$	0.096 = 9.6%
b.	6.09%	6 months (m = 2/yr)	$1.0609^{(1/2)} - 1 = 0.0300$	0.060 = 6.0%
c.	8.24%	3 months (m = 4/yr)	$1.0824^{(1/4)} - 1 = 0.0200$	0.080 = 8.0%

13. Solve the following for t: $1.08^t = 2 \Rightarrow t = 11.9$ years

    On a financial calculator, enter: PV = (–)1, FV = 2, PMT = 0, i = 6 and then compute n.

14. Semiannual compounding means that the 8.6 percent loan really carries interest of 4.3 percent per half year. Similarly, the 8.4 percent loan has a *monthly* rate of 0.7 percent.

APR	Compounding period	Effective annual rate
8.6%	6 months (m = 2/yr)	$1.043^2 - 1 = 0.0878 = 8.78\%$
8.4%	1 month (m = 12/yr)	$1.007^{12} - 1 = 0.0873 = 8.73\%$

Choose the 8.4 percent loan for its slightly lower effective rate.

15. APR = 1% × 52 = 52%

EAR = $(1.01)^{52} - 1 = 0.6777 = 67.77\%$

16. Since we are assuming that it is currently 2007, 107 years have passed since 1900.

a. $\$1{,}000 \times (1.05)^{107} = \$185{,}035.50$

b. $PV \times (1.05)^{107} = \$1{,}000{,}000 \Rightarrow PV = \$5{,}404.37$

17. $\$1{,}000 \times 1.04 = \$1{,}040.00 \Rightarrow$ interest = \$40

$\$1{,}040 \times 1.04 = \$1{,}081.60 \Rightarrow$ interest = \$1,081.60 – \$1,040 = \$41.60

After 9 years, your account has grown to: $\$1{,}000 \times (1.04)^9 = \$1{,}423.31$

After 10 years, your account has grown to: $\$1{,}000 \times (1.04)^{10} = \$1{,}480.24$

Interest earned in tenth year = \$1,480.24 – \$1,423.31 = \$56.93

18. If you earned simple interest (without compounding), then the total growth in your account after 25 years would be: 4% per year × 25 years = 100%
Therefore, your money would double. With compound interest, your money would grow faster than it would with simple interest, and therefore would require less than 25 years to double.

19. We solve the following equation for r:

$$422.41 \times (1 + r)^{10} = 1000 \Rightarrow r = 9.00\%$$

[On a financial calculator, enter: PV = (–)422.41, FV = 1000, n = 10, PMT = 0, and compute the interest rate.]

20. The PV for the quarterback is the present value of a 5-year, $3 million annuity:

$3 million × annuity factor(10%, 5 years) =

$$\$3\,\text{million} \times \left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^5}\right] = \$11.37\,\text{million}$$

The receiver gets $4 million now plus a 5-year, $2 million annuity. The present value of the annuity is:

$$\$2\,\text{million} \times \left[\frac{1}{0.10} - \frac{1}{0.10(1.10)^5}\right] = \$7.58\,\text{million}$$

With the $4 million immediate payment, the receiver's contract is worth:

$4 million + $7.58 million = $11.58 million

The receiver's contract is worth more than the quarterback's even though the receiver's *undiscounted* total payments are less than the quarterback's.

21. Rate of growth for apples: $\$0.93 \times (1 + r)^{10} = \$1.18 \Rightarrow r = 2.41\%$

Rate of growth for oranges: $\$0.96 \times (1 + r)^{10} = \$1.50 \Rightarrow r = 4.56\%$

Price of apples in 2024: $\$1.18 \times (1.0241)^{20} = \$1.90$

Price of oranges in 2024: $\$1.50 \times (1.0456)^{20} = \$3.66$

22. If the payment is denoted C, then:

$$C \times \left[\frac{1}{(0.10/12)} - \frac{1}{(0.10/12) \times [1 + (0.10/12)]^{48}}\right] = \$8{,}000 \Rightarrow C = PMT = \$202.90$$

The monthly interest rate is: 0.10/12 = 0.008333 = 0.8333 percent

Therefore, the effective annual interest rate on the loan is:

$(1.008333)^{12} - 1 = 0.1047 = 10.47$ percent

23. a. PV = 100 × annuity factor(6%, 3 periods) = $100 \times \left[\frac{1}{0.06} - \frac{1}{0.06(1.06)^3}\right] = \$267.30$

b. If the payment stream is deferred by an additional year, then each payment is discounted by an additional factor of 1.06. Therefore, the present value is reduced by a factor of 1.06 to: $267.30/1.06 = $252.17

24. a. This is an annuity problem; use trial-and-error to solve for r in the following equation:

$$\$600 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{240}}\right] = \$80{,}000 \Rightarrow r = 0.548\%$$

Using a financial calculator, enter: PV = (-)80,000, n = 20 × 12 = 240 months FV = 0, PMT = 600, compute i. To compute EAR:

$$EAR = (1 + 0.00548)^{12} - 1 = 0.0678 = 6.78\%$$

b. Compute the payment by solving for C in the following equation:

$$C \times \left[\frac{1}{0.005} - \frac{1}{0.005 \times (1.005)^{240}}\right] = \$80{,}000 \Rightarrow C = PMT = \$573.14$$

Using a financial calculator, enter: n = 240, i = 0.5%, FV = 0, PV = (–)80,000 and compute PMT = $573.14

25. a. Your monthly payments of $400 can support a loan of: $15,189.58
This is computed as follows:

$$PV = \$400 \times \left[\frac{1}{0.01} - \frac{1}{0.01 \times (1.01)^{48}}\right] = \$15{,}189.58$$

Using a financial calculator, enter: n = 48, i = 12%/12 = 1%, FV = 0, PMT = 400 and compute PV = $15,189.58

With a down payment of $2,000, you can pay at most $17,189.58 for the car.

b. In this case, n increases from 48 to 60. You can take out a loan of $17,982.02 based on this payment schedule. This is computed as follows:

$$PV = \$400 \times \left[\frac{1}{0.01} - \frac{1}{0.01 \times (1.01)^{60}}\right] = \$17{,}982.02$$

Thus, you can pay $19,982.02 for the car.

26. a. With PV = $9,000 and FV = $10,000, the annual interest rate is determined by solving the following equation for r:

$$\$9{,}000 \times (1 + r) = \$10{,}000 \Rightarrow r = 11.11\%$$

b. The present value is: $10,000 × (1 – d)

The future value to be paid back is $10,000.

Therefore, the annual interest rate is determined as follows:

$$PV \times (1 + r) = FV$$

$$[\$10{,}000 \times (1 - d)] \times (1 + r) = \$10{,}000$$

$$1 + r = \frac{1}{1-d} \Rightarrow r = \frac{1}{1-d} - 1 = \frac{d}{1-d} > d$$

c. The discount is calculated as a fraction of the future value of the loan. In fact, the proper way to compute the interest rate is as a fraction of the funds borrowed. Since PV is less than FV, the interest payment is a smaller fraction of the future value of the loan than it is of the present value. Thus, the true interest rate exceeds the stated discount factor of the loan.

27. a. If we assume cash flows come at the end of each period (ordinary annuity) when in fact they actually come at the beginning (annuity due), we discount each cash flow by one period too many. Therefore we can obtain the PV of an annuity due by multiplying the PV of an ordinary annuity by (1 + r).

b. Similarly, the FV of an annuity due equals the FV of an ordinary annuity times (1 + r). Because each cash flow comes at the beginning of the period, it has an extra period to earn interest compared to an ordinary annuity.

28. Use trial-and-error to solve the following equation for r:

$$\$240 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{48}}\right] = \$8{,}000 \Rightarrow r = 1.599\%$$

Using a financial calculator, enter: PV = (–)8000; n = 48; PMT = 240; FV = 0, then compute r = 1.599% per month.

APR = 1.599 % × 12 = 19.188%

The effective annual rate is: $(1.01599)^{12} - 1 = 0.2097 = 20.97\%$

29. The annual payment over a four-year period that has a present value of $8,000 is computed by solving the following equation for C:

$$C \times \left[\frac{1}{0.2097} - \frac{1}{0.2097 \times (1.2097)^4}\right] = \$8{,}000 \Rightarrow C = \text{PMT} = \$3{,}147.29$$

[Using a financial calculator, enter: PV = (–)8000, n = 4, FV = 0, i = 20.97, and compute PMT.] With monthly payments, you would pay only $240 × 12 = $2,880 per year. This value is lower because the monthly payments come before year-end, and therefore have a higher PV.

30. Leasing the truck means that the firm must make a series of payments in the form of an annuity. Calculate the present value as follows:

$$\text{PV} = \$8{,}000 \times \left[\frac{1}{0.07} - \frac{1}{0.07 \times (1.07)^6}\right] = \$38{,}132.32$$

Using a financial calculator, enter: PMT = 8,000, n = 6, i = 7%, FV = 0, and compute PV = $38,132.32

Since $38,132.32 < $40,000 (the cost of buying a truck), it is less expensive to lease than to buy.

31. PV of an annuity due = PV of ordinary annuity × (1 + r)

(See problem 27 for a discussion of the value of an ordinary annuity versus an annuity due.) Therefore, with immediate payment, the value of the lease payments increases from $38,132.32 (as shown in the previous problem) to:

$38,132.32 × 1.07 = $40,801.58

Since this is greater than $40,000 (the cost of buying a truck), we conclude that, if the first payment on the lease is due immediately, it is less expensive to buy the truck than to lease it.

32. Compare the present value of the payments. Assume the product sells for $100.

Installment plan:

PV = $25 + [$25 × annuity factor(5%, 3 years)]

$$\text{PV} = \$25 + \$25 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^3}\right] = \$93.08$$

Pay in full: Payment net of discount = $90

Choose the second payment plan for its lower present value of payments.

33. Installment plan:

$$PV = \$25 \times \text{annuity factor}(5\%, 4 \text{ years})$$

$$PV = \$25 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^4}\right] = \$88.65$$

Now the installment plan offers the lower present value of payments.

34. a. Solve for C in the following equation:

$$C \times \text{annuity factor}(12\%, 5 \text{ years}) = \$1{,}000$$

$$C \times \left[\frac{1}{0.12} - \frac{1}{0.12 \times (1.12)^5}\right] = \$1{,}000$$

$$C \times 3.6048 = \$1{,}000 \Rightarrow C = PMT = \$277.41$$

b. If the first payment is made immediately instead of in a year, the annuity factor will be greater by a factor of 1.12. Therefore:

$$C \times (3.6048 \times 1.12) = \$1{,}000 \Rightarrow C = PMT = \$247.69$$

35. This problem can be approached in two steps. First, find the present value of the \$10,000, 10-year annuity as of year 3, when the first payment is exactly one year away (and is therefore an ordinary annuity). Then discount the value back to today.

(1) $$PV_3 = \$10{,}000 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{10}}\right] = \$77{,}217.35$$

[Using a financial calculator, enter: PMT = 10,000; FV = 0; n = 10; i = 5%, and compute PV_3 = \$77,217.35]

(2) $$PV_0 = \frac{PV_3}{(1+r)^3} = \frac{\$77{,}217.35}{1.05^3} = \$66{,}703.25$$

36. The monthly payment is based on a \$100,000 loan:

$$C \times \left[\frac{1}{0.01} - \frac{1}{0.01 \times (1.01)^{360}}\right] = \$100{,}000 \Rightarrow C = PMT = \$1{,}028.61$$

The net amount received is \$98,000. Therefore:

$$\$1{,}028.61 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{360}}\right] = \$98{,}000 \Rightarrow r = 1.023\% \text{ per month}$$

The effective rate is: $(1.01023)^{12} - 1 = 0.1299 = 12.99\%$

37. The payment on the mortgage is computed as follows:

$$C \times \left[\frac{1}{(0.06/12)} - \frac{1}{(0.06/12) \times [1 + (0.06/12)]^{360}} \right] = \$100{,}000 \Rightarrow C = \text{PMT} = \$599.55$$

After 12 years, 216 months remain on the loan, so the loan balance is:

$$\$599.55 \times \left[\frac{1}{(0.06/12)} - \frac{1}{(0.06/12) \times [1 + (0.06/12)]^{216}} \right] = \$79{,}079.37$$

38. a. $$C \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^4} \right] = \$1{,}000 \Rightarrow C = \text{PMT} = \$301.92$$

Using a financial calculator, enter: PV = (–)1,000, FV = 0, i = 8%, n = 4, and compute PMT = $301.92

b.

Time	Loan balance	Year-end interest due	Year-end payment	Amortization of loan
0	$1,000.00	$80.00	$301.92	$221.92
1	$778.08	$62.25	$301.92	$239.67
2	$538.41	$43.07	$301.92	$258.85
3	$279.56	$22.36	$301.92	$279.56
4	$ 0.00	$ 0.00	--	--

c. $$\text{PV} = \$301.92 \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^3} \right] = \$778.08$$

Therefore, the loan balance is $778.08 after one year.

39. The loan repayment is an annuity with present value equal to $4,248.68. Payments are made monthly, and the monthly interest rate is 1%. We need to equate this expression to the amount borrowed ($4,248.68) and solve for the number of months (t).

$$\$200 \times \left[\frac{1}{0.01} - \frac{1}{0.01 \times (1.01)^t} \right] = \$4{,}248.68 \Rightarrow t = 24 \text{ months, or 2 years}$$

Using a financial calculator, enter: PV = (–)4248.68, FV = 0, i = 1%, PMT = 200, and compute n = 24.

The effective annual rate on the loan is: $(1.01)^{12} - 1 = 0.1268 = 12.68\%$

40. The present value of the $2 million, 20-year annuity, discounted at 8%, is:

$$\text{PV} = \$2\text{ million} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{20}}\right] = \$19.64\text{ million}$$

If the payment comes immediately, the present value increases by a factor of 1.08 to $21.21 million.

41. The real rate is zero. With a zero real rate, we simply divide her savings by the years of retirement: $450,000/30 = $15,000 per year

42. r = 0.5% per month

$\$1{,}000 \times (1.005)^{12} = \$1{,}061.68$

$\$1{,}000 \times (1.005)^{18} = \$1{,}093.93$

43. You are repaying the loan with payments in the form of an annuity. The present value of those payments must equal $100,000. Therefore:

$$\$804.62 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{360}}\right] = \$100{,}000 \Rightarrow r = 0.750\%\text{ per month}$$

[Using a financial calculator, enter: PV = (–)100,000, FV = 0, n = 360, PMT = 804.62, and compute the interest rate.]

The effective annual rate is: $(1.00750)^{12} - 1 = 0.0938 = 9.38\%$

The lender is more likely to quote the APR (0.750% × 12 = 9%), which is lower.

44. EAR = $e^{0.06} - 1 = 1.0618 - 1 = 0.0618 = 6.18\%$

45. The present value of the payments for option (a) is $22,000.
The present value of the payments for option (b) is:

$$\text{PV} = \$500 \times \left[\frac{1}{0.01} - \frac{1}{0.01 \times (1.01)^{48}}\right] = \$18{,}986.98$$

Option (b) is the better deal.

46. $\$100 \times e^{0.10 \times 8} = \$222.55$

$\$100 \times e^{0.08 \times 10} = \$222.55$

47 Your savings goal is FV = \$30,000. You currently have in the bank PV = \$20,000. Solve the following equation for t:

$$(\$20{,}000 \times 1.005^{t}) + \$100 \times \left[\frac{1.005^{t} - 1}{0.005}\right] = \$30{,}000 \Rightarrow t = 44.74 \text{ months}$$

Using a financial calculator, enter FV=30000, PV=(–)20000, PMT = (–)100 and r = 0.5%. Solve for n to find n = 44.74 months.

48. The present value of your payments to the bank equals:

$$\text{PV} = \$100 \times \left[\frac{1}{0.06} - \frac{1}{0.06 \times (1.06)^{10}}\right] = \$736.01$$

The present value of your receipts is the value of a \$100 perpetuity deferred for 10 years:

$$\frac{100}{0.06} \times \frac{1}{(1.06)^{10}} = \$930.66$$

This is a good deal if you can earn 6% on your other investments.

49. If you live forever, you will receive a \$100 perpetuity that has present value equal to: \$100/r

Therefore: \$100/r = \$2500 ⇒ r = 4 percent

50. r = \$10,000/\$125,000 = 0.08 = 8 percent

51. a. The present value of the ultimate sales price is: \$4 million/$(1.08)^5$ = \$2.722 million

b. The present value of the sales price is less than the cost of the property, so this would not be an attractive opportunity.

c. The present value of the total cash flows from the property is now:

PV = [\$0.2 million × annuity factor(8%, 5 years)] + \$4 million/$(1.08)^5$

$$= \$0.2 \text{ million} \times \left[\frac{1}{0.08} - \frac{1}{0.08 \times (1.08)^{5}}\right] + \frac{\$4 \text{ million}}{(1.08)^{5}} =$$

= \$0.799 million + \$2.722 million = \$3.521 million

Therefore, the property is an attractive investment if you can buy it for \$3 million.

52. PV of cash flows = ($120,000/1.12) + ($180,000/1.12^2) + ($300,000/$1.12^3$) = $464,171.83

This exceeds the cost of the factory, so the investment is attractive.

53. a. The present value of the future payoff is: $2,000/$(1.06)^{10}$ = $1,116.79

This is a good deal: present value exceeds the initial investment.

b. The present value is now equal to: $2,000/$(1.10)^{10}$ = $771.09

This is now less than the initial investment. Therefore, this is a bad deal.

54. Suppose the purchase price is $1. If you pay today, you get the discount and pay only $0.97. If you wait a month, you pay $1. Thus, you can view the deferred payment as saving a cash flow of $0.97 today, but paying $1 in a month. Therefore, the monthly rate is:

0.03/0.97 = 0.0309 = 3.09%

The effective annual rate is: $(1.0309)^{12} - 1 = 0.4408$ = 44.08%

55. You borrow $1,000 and repay the loan by making 12 monthly payments of $100. Solve for r in the following equation:

$$\$100 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{12}}\right] = \$1{,}000 \Rightarrow r = 2.923\% \text{ per month}$$

[Using a financial calculator, enter: PV = (–)1,000, FV = 0, n = 12, PMT = 100, and compute r = 2.923%]

Therefore, the APR is: 2.923% × 12 = 35.076%

The effective annual rate is: $(1.02923)^{12} - 1 = 0.41302$ = 41.302%

If you borrowed $1,000 today and paid back $1,200 one year from today, the true rate would be 20%. You should have known that the true rate must be greater than 20% because the twelve $100 payments are made before the end of the year, thus increasing the true rate above 20%.

56. You will have to pay back the original \$1,000 plus $(3 \times 20\%) = 60\%$ of the loan amount, or \$1600 over the three years. This implies monthly payments of:

$$\$1{,}600/36 = \$44.44$$

The monthly interest rate is obtained by solving:

$$\$44.44 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{36}}\right] = \$1{,}000 \Rightarrow r = 2.799\% \text{ per month}$$

Using a financial calculator, enter: PV = (–)1,000, FV = 0, n = 36, PMT = 44.44, and compute r = 2.799%

Therefore, the APR is: $2.799\% \times 12 = 33.588\%$

The effective annual rate is: $(1.02799)^{12} - 1 = 0.39273 = 39.273\%$

57. For every \$1,000 borrowed, the present value is: $[\$1{,}000 \times (1 - d)]$

The future value to be paid back is \$1,000. Therefore, the annual interest rate is determined as follows:

$$PV \times (1 + r) = FV$$

$$[\$1{,}000 \times (1 - d)] \times (1 + r) = \$1{,}000$$

$$1 + r = \frac{1}{1-d} \Rightarrow r = \frac{1}{1-d} - 1 = \frac{d}{1-d} > d$$

If d = 20%, then the effective annual interest rate is: $(0.2/0.8) = 0.25 = 25\%$

58. After one year, each dollar invested at First National will grow to:

$$\$1 \times (1.031)^2 = \$1.06296$$

After one year, each dollar invested at Second National will grow to:

$$\$1 \times (1.005)^{12} = \$1.06168$$

First National pays the higher effective annual rate.

59. Since the \$20 initiation fee is taken out of the proceeds of the loan, the amount actually borrowed is: \$1,000 – \$20 = \$980

The monthly rate is found by solving the following equation for r:

$$\$90 \times \left[\frac{1}{r} - \frac{1}{r \times (1+r)^{12}}\right] = \$980 \Rightarrow r = 1.527\% \text{ per month}$$

The effective rate is: $(1.01527)^{12} - 1 = 0.1994 = 19.94\%$

60. The future value of the payments into your savings fund must accumulate to \$500,000. We choose the payment (C) so that:

C × future value of an annuity = \$500,000

$$C \times \left[\frac{1.06^{40} - 1}{0.06}\right] = \$500{,}000 \Rightarrow C = PMT = \$3{,}230.77$$

Using a financial calculator, enter: n = 40; i = 6; PV = 0; FV = 500,000, compute PMT = \$3,230.77

61. If you invest the \$100,000 received in year 10 until your retirement in year 40, it will grow to: \$100,000 × $(1.06)^{30}$ = \$574,349.12

Therefore, you do not need any additional savings; investing the \$100,000 produces a future value that exceeds your \$500,000 requirement.

62. By the time you retire you will need:

$$PV = \$40{,}000 \times \left[\frac{1}{0.06} - \frac{1}{0.06 \times (1.06)^{20}}\right] = \$458{,}796.85$$

The future value of the payments into your savings fund must accumulate to: \$458,796.85 We choose the payment (C) so that:

C × future value of an annuity = \$458,796.85

$$C \times \left[\frac{1.06^{40} - 1}{0.06}\right] = \$458{,}796.85 \Rightarrow C = PMT = \$2{,}964.53$$

Using a financial calculator, enter: n = 40; i = 6; PV = 0; FV = 458,796.85 and compute PMT = \$2,964.53

63. a. After 30 years, the couple will have accumulated the future value of a \$3,000 annuity, plus the future value of the \$10,000 gift. The sum of the savings from these sources is:

$$\$3{,}000 \times \left[\frac{1.08^{30} - 1}{0.08}\right] + (\$10{,}000 \times 1.08^{25}) =$$

\$339,849.63 + \$68,484.75 = \$408,334.38

b. If they wish to accumulate $800,000 by retirement, they have to save an *additional* amount *per year* to provide additional accumulations of: $391,665.62 This requires additional annual savings of:

$$C\times\left[\frac{1.08^{30}-1}{0.08}\right]=\$391{,}665.62\Rightarrow C=PMT=\$3{,}457.40$$

[Using a financial calculator, enter: i = 8; n = 30; PV = 0; FV = 391,665.62 and compute PMT.]

64. a. The *present value* of the planned consumption stream *as of the retirement date* will be:

$$PV=\$30{,}000\times\left[\frac{1}{0.08}-\frac{1}{0.08\times(1.08)^{25}}\right]=\$320{,}243.29$$

Therefore, they need to have accumulated this amount of savings by the time they retire. So, their savings plan must provide a *future value* of: $320,243.29
With 50 years to save at 8%, the savings annuity must be:

$$C\times\left[\frac{1.08^{50}-1}{0.08}\right]=\$320{,}243.29\Rightarrow C=PMT=\$558.14$$

Another way to think about this is to recognize that the present value of the savings stream must equal the present value of the consumption stream. The PV of consumption as of today is: $\$320{,}243.29/(1.08)^{50} = \$6{,}827.98$

Therefore, we set the *present* value of savings equal to this value, and solve for the required savings stream.

b. The couple needs to accumulate additional savings with a present value of:

$\$60{,}000/(1.08)^{20} = \$12{,}872.89$

The total PV of savings is now: $12,872.89 + $6,827.98 = $19,700.87

Now we solve for the required savings stream as follows:

$$C\times\left[\frac{1}{0.08}-\frac{1}{0.08\times(1.08)^{50}}\right]=\$19{,}700.87\Rightarrow C=PMT=\$1{,}610.41$$

Using a financial calculator, enter: n = 50; i = 8; PV = (–)19,700.87; FV = 0; and then compute PMT = $1,610.41

65. $60,000/6.9 = $8,696. Her real income increased from $6,000 to $8,696.

66. (1 + nominal interest rate) = (1 + real interest rate) $\times$ (1 + inflation rate)

a. $1.03 \times 1.0 = 1.03 \Rightarrow$ nominal interest rate = 3.00%

b. $1.03 \times 1.04 = 1.0712 \Rightarrow$ nominal interest rate = 7.12%

c. $1.03\times 1.06 = 1.0918 \Rightarrow$ nominal interest rate = 9.18%

67. $\text{real interest rate} = \dfrac{1+\text{nominal interest rate}}{1+\text{inflation rate}} - 1$

a. (1.06/1) – 1 = 0.0600 = 6.00%

b. (1.06/1.03) – 1 = 0.0291 = 2.91%

c. (1.06/1.06) – 1 = 0.0%

68. a. PV = $\$100/(1.08)^3$ = \$79.38

b. real value = $\$100/(1.03)^3$ = \$91.51

c. $\text{real interest rate} = \dfrac{1+\text{nominal interest rate}}{1+\text{inflation rate}} - 1$ = 0.04854 = 4.854%

d. PV = $\$91.51/(1.04854)^3$ = \$79.38

69. a. The real interest rate is: (1.06/1.02) – 1 = 3.92%

Therefore, the present value is:

$$\text{PV} = \$100{,}000 \times \left[\frac{1}{0.0392} - \frac{1}{0.0392 \times (1.0392)^5}\right] = \$446{,}184.51$$

b. If cash flow is level in nominal terms, use the 6% nominal interest rate to discount. The annuity factor is now 4.21236 and the cash flow stream is worth only \$421,236.

70. a. \$1 million will have a real value of:\$1 million/$(1.03)^{50}$ = \$228,107

b. At a real rate of 2%, this can support a real annuity of:

$$C \times \left[\frac{1}{0.02} - \frac{1}{0.02 \times (1.02)^{20}}\right] = \$228{,}107 \Rightarrow C = \text{PMT} = \$13{,}950$$

[To solve this on a financial calculator, enter: n = 20, i = 2, PV = 228,107, FV = 0, and then compute PMT.]

71. According to the Rule of 72, at an interest rate of 6%, it will take 72/6 = 12 years for your money to double. For it to quadruple, your money must double, and then double again. This will take approximately 24 years.

Using a financial calculator, enter: i = 6, PV = (–)1, FV = , and then compute n = 23.79 years.

The real interest rate is: (1.06/1.04) – 1 = 0.0192 = 1.92%

Purchasing power increases by: $(1.0192)^{24} - 1 = 0.5784 = 57.84\%$

72. $(1+1.11)^{12} - 1 = 7786.37 = 778{,}637\%\%$

Prices increased by 778,637 percent per year.

73. Using the perpetuity formula, the 4% perpetuity will sell for: £4/0.06 = £66.67

The 2½% perpetuity will sell for: £2.50/0.06 = £41.67

74. a. $$\text{PV} = \$30{,}000 \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^{15}}\right] = \$228{,}182.39$$

b. The present value of the retirement goal is:

$$\$228{,}182.39/(1.10)^{30} = \$13{,}076.80$$

The present value of your 30-year savings stream must equal this present value. Therefore, we need to find the payment for which:

$$C \times \left[\frac{1}{0.10} - \frac{1}{0.10 \times (1.10)^{30}}\right] = \$13{,}076.80 \Rightarrow C = \text{PMT} = \$1{,}387.18$$

You must save $1,387.18 per year.

c. $1.00 \times (1.04)^{30} = \$3.24$

d. We repeat part (a) using the real interest rate: (1.10/1.04) – 1 = 0.0577 or 5.77%

The retirement goal in real terms is:

$$\text{PV} = \$30{,}000 \times \left[\frac{1}{0.0577} - \frac{1}{0.0577 \times (1.0577)^{15}}\right] = \$295{,}796.61$$

e. The future value of your 30-year savings stream must equal: $295,796.61 Therefore, we solve for payment (PMT) in the following equation:

$$C \times \left[\frac{1.0577^{30} - 1}{0.0577} \right] = \$295{,}796.61 \Rightarrow C = PMT = \$3{,}895.66$$

Therefore, we find that you must save $3,895.66 per year in real terms. This value is much higher than the result found in part (b) because the rate at which purchasing power grows is less than the nominal interest rate, 10%.

f. If the *real* amount saved is $3,895.66 and prices rise at 4 percent per year, then the amount saved at the end of one year, in nominal terms, will be:

$\$3{,}895.66 \times 1.04 = \$4{,}051.49$

The thirtieth year will require nominal savings of:

$3{,}895.66 \times (1.04)^{30} = \$12{,}635.17$

75. a. We redo problem 64, but now we use the real interest rate, which is:

$(1.08/1.04) - 1 = 0.0385 = 3.85\%$

We note that the $30,000 expenditure stream now must be interpreted as a real annuity, which will rise along with the general level of prices at the inflation rate of 4%.

We find that the PV of the required real savings stream *as of the retirement date* is:

$$PV = \$30{,}000 \times \left[\frac{1}{0.0385} - \frac{1}{0.0385 \times (1.0385)^{25}} \right] = \$476{,}182.14$$

[Using a financial calculator, enter: n = 25; i = 3.85; FV = 0; PMT = 30,000 and compute PV.]

This requires a savings stream with a real future value of $476,182.14, which means that the real savings stream must be: $3,266.82

$$C \times \left[\frac{1.0385^{50} - 1}{0.0385} \right] = \$476{,}182.14 \Rightarrow C = PMT = \$3{,}266.82$$

[Using a financial calculator, enter: n = 50; i = 3.85; FV = (–)476,182.14; and then compute PMT.]

b. Nominal savings in year one will be: $\$3{,}266.82 \times 1.04 = \$3{,}397.49$

c. Nominal savings in the last year will be: $\$3{,}266.82 \times (1.04)^{50} = \$23{,}216.26$

d. Nominal expenditures in the first year of retirement will be:

$\$30{,}000 \times (1.04)^{51} = \$221{,}728.52$

Nominal expenditures in the last year of retirement will be:

$\$30{,}000 \times (1.04)^{75} = \$568{,}357.64$

76. The interest rate per three months is: 6%/4 = 1.5%

Therefore, the value of the perpetuity is: $100/0.015 = $6,666.67

77. $FV = PV \times (1 + r_0) \times (1 + r_1) = \$1 \times 1.08 \times 1.10 = \$1.188$

$$PV = \frac{FV}{(1+r_0)\times(1+r_1)} = \frac{\$1}{1.08\times1.10} = \$0.8418$$

78. You earned compound interest of 6% for 8 years and 4% for 13 years. Your $1,000 has grown to:

$\$1{,}000 \times (1.06)^8 \times (1.04)^{13} = \$2{,}653.87$

79. a. First, calculate the present value of all lifetime expenditures

General living expenses of $50,000 per year for 50 years:

$$\$50{,}000\times\left[\frac{1}{0.05}-\frac{1}{0.05\times(1.05)^{50}}\right]=\$912{,}796$$

Apartment rental of $16,000 for 8 years

$$\$16{,}000\times\left[\frac{1}{0.05}-\frac{1}{0.05\times(1.05)^{8}}\right]=\$103{,}411$$

Home purchase of $250,000 in 9 years

$$PV = \$250{,}000/(1.05)^9 = \$161{,}152$$

Five automobile purchases of $30,000 in each of years 0, 10, 20, 30, 40, and 50.

$$PV = \$30{,}000/(1.05)^0 = \$30{,}000$$
$$PV = \$30{,}000/(1.05)^{10} = \$18{,}417$$

$$PV = \$30{,}000/(1.05)^{20} = \$11{,}307$$
$$PV = \$30{,}000/(1.05)^{30} = \$6{,}941$$
$$PV = \$30{,}000/(1.05)^{40} = \$4{,}261$$
$$PV = \$30{,}000/(1.05)^{50} = \$2{,}616$$

College education of $150,000 in 25 years

$$PV = \$150{,}000/(1.05)^{25} = \$44{,}295$$

College education of $150,000 in 30 years

$$PV = \$150{,}000/(1.05)^{30} = \$34{,}707$$

Retirement Portfolio of $436,177 in 50 years ($436,177 = PV of a 20 year annuity paying $35,000).

$$\$35{,}000 \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{20}} \right] = \$436{,}177$$

$$PV = \$436{,}177/(1.05)^{50} = \$38{,}036$$

Summing the present value of all lifetime expenditures gives $1,367,939 = 912,796 + 103,411 + 161,152 + 30,000 + 18,417 + 11,307 + 6,941 + 4,261 + 2,616 + 44,295 + 34,707 + 38,036.

To find the average salary necessary to support this lifetime consumption plan we solve for the 50 year payment with the same present value:

$$C \times \left[\frac{1}{0.05} - \frac{1}{0.05 \times (1.05)^{50}} \right] = \$1{,}367{,}939 \Rightarrow C = PMT = \$74{,}931$$

b. In part a we have discounted real cash flows using a nominal interest rate. Here we repeat the process using a real interest rate of 1.94% (0.0194 = 1.05/1.03 – 1).

General living expenses of $50,000 per year for 50 years:

$$\$50{,}000 \times \left[\frac{1}{0.0194} - \frac{1}{0.00194 \times (1.0194)^{50}} \right] = \$1{,}591{,}184$$

Apartment rental of $16,000 for 8 years

$$\$16{,}000 \times \left[\frac{1}{0.0194} - \frac{1}{0.00194 \times (1.0194)^{8}} \right] = \$117{,}511$$

Home purchase of $250,000 in 9 years

$$PV = \$250{,}000/(1.00194)^9 = \$210{,}300$$

Five automobile purchases of $30,000 in each of years 0, 10, 20, 30, 40, and 50.

$$PV = \$30{,}000/(1.0194)^0 = \$30{,}000$$
$$PV = \$30{,}000/(1.0194)^{10} = \$24{,}756$$
$$PV = \$30{,}000/(1.0194)^{20} = \$20{,}428$$
$$PV = \$30{,}000/(1.0194)^{30} = \$16{,}857$$
$$PV = \$30{,}000/(1.0194)^{40} = \$13{,}910$$
$$PV = \$30{,}000/(1.0194)^{50} = \$11{,}479$$

College education of $150,000 in 25 years

$$PV = \$150{,}000/(1.0194)^{25} = \$92{,}785$$

College education of $150,000 in 30 years

$$PV = \$150{,}000/(1.0194)^{30} = \$84{,}285$$

Retirement Portfolio of $575,530 in 50 years ($575,530 = PV of a 20 year annuity paying $35,000).

$$\$35{,}000 \times \left[\frac{1}{0.0194} - \frac{1}{0.0194 \times (1.0194)^{20}}\right] = \$575{,}530$$

$$PV = \$575{,}530/(1.00194)^{50} = \$220{,}210$$

Summing the present value of all lifetime expenditures gives $2,433,705 = 1,591,184 + 117,511 + 210,300 + 30,000 + 24,756 + 20,428 + 16,857 + 13,910 + 11,479 + 92,785 + 84,285 + 220,210.

To find the average salary necessary to support this lifetime consumption plan we solve for the 50 year payment with the same present value:

$$C \times \left[\frac{1}{0.0194} - \frac{1}{0.0194 \times (1.0194)^{50}}\right] = \$2{,}433{,}705 \Rightarrow C = PMT = \$76{,}475$$

This average real salary is equivalent to the salary from part a, $74,931, growing to $135,334 in just 20 years ($74,931 x $(1 + 0.03)^{20}$ = $135,334).

With these expected lifetime expenditures a 2010 graduate must “save” on average $76,475 each year; a challenge, given that the required annual savings will grow along with other prices each year.

80. a. Using Table 5-4, the annuity factor is 11.4699. The annual payment on the loan is therefore \$100,000/11.4699 = \$8,718.47.

b.

Year	Beginning-of-Year Balance	Year-End Interest Due on Balance	Year-End Payment	Amortization of Loan	End-of-Year Balance
1	100,000.00	6,000	8,718.46	2,718.46	97,281.54
2	97,281.54	5,837	8,718.46	2,881.56	94,399.98
3	94,399.98	5,664	8,718.46	3,054.46	91,345.52
4	91,345.52	5,481	8,718.46	3,237.72	88,107.80
5	88,107.80	5,286	8,718.46	3,431.99	84,675.81
6	84,675.81	5,081	8,718.46	3,637.91	81,037.91
7	81,037.91	4,862	8,718.46	3,856.18	77,181.72
8	77,181.72	4,631	8,718.46	4,087.55	73,094.17
9	73,094.17	4,386	8,718.46	4,332.81	68,761.37
10	68,761.37	4,126	8,718.46	4,592.77	64,168.59
11	64,168.59	3,850	8,718.46	4,868.34	59,300.25
12	59,300.25	3,558	8,718.46	5,160.44	54,139.81
13	54,139.81	3,248	8,718.46	5,470.07	48,669.75
14	48,669.75	2,920	8,718.46	5,798.27	42,871.47
15	42,871.47	2,572	8,718.46	6,146.17	36,725.31
16	36,725.31	2,204	8,718.46	6,514.94	30,210.37
17	30,210.37	1,813	8,718.46	6,905.83	23,304.54
18	23,304.54	1,398	8,718.46	7,320.18	15,984.35
19	15,984.35	959	8,718.46	7,759.39	8,224.96
20	8,224.96	493	8,718.46	8,224.96	0.00

c. The initial loan payment is 6,000 of 8,718.46, or 69%. Amortization is 31%. The last loan payment is 493 of 8,718.46, or 6%. After 10 years \$35,831.41 has been paid off, or 36% of the loan.

d. If the inflation rate is 2% the real interest rate on the loan is approximately 4%. The real value of the first payment is $\$8,718.46/(1+.04)^{1}$, or \$8,383.13. The real value of the last payment is $\$8,718.46/(1+.04)^{20}$, or \$3,978.99.

e. If the inflation rate is 8% and the real interest rate is unchanged the nominal interest rate is approximately, 12%.

Year	Beginning-of-Year Balance	Year-End Interest Due on Balance	Year-End Payment	Amortization of Loan	End-of-Year Balance
1	100,000.00	12,000	13,387.88	1,387.88	98,612.12
2	98,612.12	11,833	13,387.88	1,554.42	97,057.70
3	97,057.70	11,647	13,387.88	1,740.95	95,316.74
4	95,316.74	11,438	13,387.88	1,949.87	93,366.88
5	93,366.88	11,204	13,387.88	2,183.85	91,183.02
6	91,183.02	10,942	13,387.88	2,445.92	88,737.11
7	88,737.11	10,648	13,387.88	2,739.43	85,997.68
8	85,997.68	10,320	13,387.88	3,068.16	82,929.53
9	82,929.53	9,952	13,387.88	3,436.33	79,493.19
10	79,493.19	9,539	13,387.88	3,848.70	75,644.50
11	75,644.50	9,077	13,387.88	4,310.54	71,333.96
12	71,333.96	8,560	13,387.88	4,827.80	66,506.16
13	66,506.16	7,981	13,387.88	5,407.14	61,099.02
14	61,099.02	7,332	13,387.88	6,056.00	55,043.02
15	55,043.02	6,605	13,387.88	6,782.72	48,260.30
16	48,260.30	5,791	13,387.88	7,596.64	40,663.66
17	40,663.66	4,880	13,387.88	8,508.24	32,155.42
18	32,155.42	3,859	13,387.88	9,529.23	22,626.20
19	22,626.20	2,715	13,387.88	10,672.73	11,953.46
20	11,953.46	1,434	13,387.88	11,953.46	0.00

The real value of the first payment is $13,387.88/(1+.04)^1, or $12,872.96. The real value of the last payment is $13,387.88/(1+.04)^{20}, or $6,110.05.

f. High inflation hurts the real estate market by increasing the real costs of the homeownership.

Solution to Minicase for Chapter 5

How much can Mr. Road spend each year? First let's see what happens if we ignore inflation.

1. Account for Social Security income of $750 per month, or $9,000 annually.

2. Account for the income from the savings account. Because Mr. Road does not want to run down the balance of this account, he can spend only the interest income:

   $0.05 \times \$12{,}000 = \$600$ annually

3. Compute the annual consumption available from his investment account. We find the 20-year annuity with present value equal to the value in the account:

   Present Value = annual payment × 20-year annuity factor at 9% interest rate:

   $$\text{PV} = \text{annual payment} \times \left[\frac{1}{0.09} - \frac{1}{0.09 \times (1.09)^{20}}\right]$$

   $\$180{,}000 = \text{annual payment} \times 9.129 \Rightarrow \text{Annual payment} = \$180{,}000/9.129 = \$19{,}717$

   Notice that the investment account provides annual income of $19,717, which is more than the annual interest from the account. This is because Mr. Road plans to run the account down to zero by the end of his life.

So Mr. Road can spend: $19,717 + $600 + $9,000 = $29,317 per year
This is comfortably above his current living expenses, which are $2,000 per month or $24,000 annually.

The problem of course is inflation. We have mixed up real and nominal flows. The Social Security payments are tied to the consumer price index and therefore are level in *real* terms. But the annuity of $19,717 per year from the investment account and the $600 interest from the savings account are fixed in *nominal* terms, and therefore the purchasing power of these flows will steadily decline.

For example, let's look out 15 years. At 4 percent inflation, prices will increase by a factor of $(1.04)^{15} = 1.80$. Income in 15 years will therefore be as follows:

Income Source	Nominal Income	Real Income
Social security (indexed to CPI, fixed in real terms at $9,000)	$16,200	$9,000
Savings account	600	333
Investment account (fixed nominal annuity)	19,717	10,954
Total income	$36,517	$20,287

Once we recognize inflation, we see that, in fifteen years, income from the investment account will buy only a bit more than one-half of the goods it buys today.

Obviously Mr. Road needs to spend less today and put more aside for the future. Rather than spending a constant *nominal* amount out of his savings, he would probably be better off spending a constant *real* amount.

Since we are interested in level real expenditures, we must use the real interest rate to calculate the 20-year annuity that can be provided by the $180,000. The real interest rate is:

real interest rate = (1.09/1.04) − 1 = 1.048 − 1 = 4.8%

We therefore calculate the real sum that can be spent out of savings as follows:

$$C \times \left[\frac{1}{0.048} - \frac{1}{0.048 \times (1.048)^{20}} \right] = \$180{,}000 \Rightarrow C = PMT = \$14{,}200$$

[Using a financial calculator, enter: n = 20; i = 4.8; PV = (–)180,000; and then compute PMT = $14,200]

Thus Mr. Road's investment account can generate real income of $14,200 per year. The real value of Social Security is fixed at $9,000. Finally, if we assume that Mr. Road wishes to maintain the *real* value of his savings account at $12,000, then he will have to increase the balance of the account in line with inflation, that is, by 4% each year. Since the nominal interest rate on the account is 5%, only the first 1% of interest earnings on the account, or $120 real dollars, is available for spending each year. The other 4% of earnings must be re-invested. So total real income is: $14,200 + $9,000 + $120 = $23,320

To keep pace with inflation Mr. Road will have to spend 4 percent more of his savings each year. After one year of inflation, he will spend: 1.04 × $23,320 = $24,253

After two years he will spend: $(1.04)^2 \times \$23{,}320 = \$25{,}223$

The picture fifteen years out looks like this:

Income Source	Nominal Income	Real Income
Social security	$16,200	$9,000
Net income from savings account (i.e., net of reinvested interest)	216	120
Investment account (fixed *real* annuity)	25,560	14,200
Total income	$41,976	$23,320

Mr. Road's income and expenditure will nearly double in 15 years but his real income and expenditure are unchanged at $23,320.

This may be bad news for Mr. Road since his living expenses are $24,000. Do you advise him to prune his living expenses? Perhaps he should put part of his nest egg in junk bonds which offer higher *promised* interest rates, or into the stock market, which has generated higher returns on average than investment in bonds. These higher returns might support a higher real annuity -- but is Mr. Road prepared to bear the extra risks?

Should Mr. Road consume more today and risk having to sell his house if his savings are run down late in life? These issues make the planning problem even more difficult. It is clear, however, that one cannot plan for retirement without considering inflation.